# Semester V

## ABSTRACT ALGEBRA

Credit: 04
C. No.: UGM 0501

Semester Examination: 80 marks
Sessional Assessment: 20 marks

### UNIT 1

Review of sets, Concept of family of sets, Equivalence relation and fundamental theorem of equivalence classes, Partial order relation, congruence modulo relation in $\mathbb{Z}$ and residue classes.
Functions, types of functions, composition of functions and invertible functions. Examples and exercises based on these concepts.

### UNIT 2

Binary operations, definition of semigroups and groups with plenty of examples from number system, matrices and functions. Abelian and non abelian groups, finite groups. Subgroups and their examples, subgroup generated by the subset, cyclic groups and their properties. Examples and exercises on these topics.

### UNIT 3

Right and left cosets, normal subgroups and their examples, quotient groups and their examples, $Z_p$ as a quotient group, Lagrange's Theorem and its applications including Euler's Theorem and Fermat's Theorem, examples and exercises based on these topics.

### UNIT 4

Homomorphisms and isomorphisms of groups and their examples, Permutation group $S_n$ of degree $n$, even and odd permutations, computation of subgroups of $S_n$ for $n \leq 3$, exercises and examples based on these concepts.

## UNIT 5

The concept of ring, integral domain and fields with examples. Subrings of a ring, Ring homomorphisms, Isomorphisms with examples. Left ideals, right ideals and ideals of a ring. Prime and maximal ideals definition and examples only. The quotient ring and fundamental theorem of ring homomorphism. Exercises base on these concepts.

**Textbooks:-**


1. Halmos, P.R., Naive Set Theory, Martino Fine Books, 2011.
2. Lin, S.T. and Lin, Y., Ser Theory with Applications, Mancorp Pub. Co.
3. Stoll, R.R., Set Theory and Logic, Dover Publication.
4. Gopalakrishnan, N.S., University Algebra, New Age Internatinal.
5. Herstein, I.N., Topics in Algebra, John Wiley and Sons.
6. Singh, S. and Zameeruddin, Q., Modern Algebra, Vikas Publishing House Pvt. Ltd.


**Note:-**

1. Each lecture will be of one hour duration.
2. The question paper shall consist of ten questions, two questions from each unit. The candidate will be required to do five questions, selecting exactly one question from each unit.

# Semester VI

## VECTOR SPACES AND METRIC SPACES

Credit: 04
C. No.: UGM 0601

Semester Examination: 80 marks
Sessional Assessment: 20 marks

### UNIT 1

Definition and examples of vector spaces, subspaces of a vector space and the quotient space. Linear dependence and linear independence of a set of vectors. Linear span. Exercises and examples based on these concepts.

### UNIT 2

Basis and dimension of a vector space. Isomorphic vector spaces, finite and infinite dimensional vector spaces with plenty of illustrations. Dual space of a finite dimensional vector space - definition and examples. Dimension of a dual space. Exercises based on these concepts.

### UNIT 3

Linear transformation on vector space and their examples, algebra of linear transformations on a vector space, matrix representation of a linear transformation of finite dimensional vector space. Kernel and range of a linear transformation, inverse of a linear transformation on finite dimensional vector spaces. Examples and exercises based on these concepts.

### UNIT 4

**Topology of $\mathbb{R}$ :**

Denumerable and non denumerable sets and their examples. Open set and

closed set on the real line. their examples and properties. Limit points of a set. Heine Borel Theorem for closed and bounded intervals. Bolzano Weierstrass Theorem. Examples and exercises based on these concepts.

## UNIT 5

**Metric Spaces:**

Definition of metric with examples. Open sets and closed sets in metric spaces. Interior. closure and boundary of a set in a metric space. Equivalent conditions for open sets and closed sets. Convergence of sequences. Continuous maps and their characterizations. Examples and exercises based on these concepts.

**Textbooks:-**


1. Helson, H., Linear Algebra, Hindustan Book Agency.
2. Hoffman, K. and Kunze, R., Linear Algebra, Prentice-Hall Inc.
3. Krishnamurthy, V., Mainra. V. P. and Arora, J. L., An Introduction to Linear Algebra, East West Press Pvt. Ltd. New Delhi.
4. Gopalakrishnan, N.S., University Algebra, New Age International.
5. Halmos, P.R.. Finite Dimensional Vector Spaces,
6. Singh, S. and Zameeruddin, Q., Modern Algebra, Vikas Publishing House Pvt. Ltd.
7. Croom, F.H.. Principles of Topology, Cengage Learning India Pvt. Ltd.


**Note:-**

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2. The question paper shall consist of ten questions, two questions from each unit. The candidate will be required to do five questions, selecting exactly one question from each unit.