BACHELOR OF EDUCATION (B.Ed.)

Semester–II

(For the examination to held in the year 2018, 2019 & 2020)

METHODOLOGY OF TEACHING SUBJECT-I

Course No. 205 *Title :* **Teaching of Mathematics**

Credits 4 **Total Marks : 100**

Maximum Marks Internal : 40

Maximum Marks External : 60

Duration of Exam : 3 hrs

Objectives :

To enable the pupil teachers to:

- Study and to develop an understanding of the different aspects of Teaching Mathematics.
- Study and Understand the objectives of Teaching Mathematics.
- Study and Understand the Methods and Skills of Teaching Mathematics.
- Study and Understand the use of Club and the teacher's capacity making facilities in the Teaching of Mathematics.

UNIT-I

Mathematics –Structure and Knowledge

Meaning, nature and characteristics of mathematics; Processes in mathematics – mathematical reasoning, pattern recognition, algebraic thinking, geometric thinking (Van Hiele model of geometric thought), problem solving in mathematics creative thinking in mathematics. Structure of mathematics – Euclidean geometry-terms (undefined and defined terms), axioms, postulates and theorems; validation process of mathematical statements. Pedagogic content knowledge analysis for - facts, concepts, generalizations and procedures. Knowledge and teaching of Integers, Rational Number, Real Number Polynomials Quadratic Equation and congruency of Triangles.

UNIT II

Objectives and Methods of Mathematics Teaching

Objectives of teaching mathematics-disciplinary, utilitarian, cultural, social and recreational. Anderson's revised Bloom's taxonomy of instructional objectives – specifications; task analysis; Objectives of teaching arithmetic, algebra, geometry. Application of Approaches and models of teaching mathematics – Inductive - deductive approach; Analytic–synthetic approach ; Guided discovery approach; Project method; Concept Attainment Model. Pedagogic content knowledge for the trigonometry and coordinate geometry, Primary Concepts in Geometry Trigonometric Ratios of Complementary Angles, Height and Distance.

UNIT-III

Different Techniques of Teaching Mathematics

Five E model – engage, explore, express, expand, evaluate; Drill and Review Work in Mathematics; Assignment techniques; Problem solving technique Supervised study technique; Oral work in Mathematics, Application of techniques for Ratio, Proportion (equality of Ratios), Arithmetic Mean; Irrational numbers, Laws of Real Numbers & Integers and its examples'

UNIT IV

Mathematics Club& the role of Teacher

Mathematics Club: Concept Objectives, Importance, Different Activities of the Club in respect of the teaching of Circle, Partition of plane of a circle by the circle, Theorems on Circle and Chords of a Circle. Learning Teaching of Mathematics by co relating it with the Science and Geography- Area, Speed Time, Volume & surface Area.

Mathematics Teacher : Qualities and Competencies – Listening, Understanding and Expression

Note for Paper Setters

The Question paper consists of 9 questions having Q. No. 1 as Compulsory having four parts spread over the entire Syllabus, with a weightage of 12 marks. The rest of Question paper is divided into four Units and the students are to attend four Questions from these units with the internal choice. The essay type Question carries 12 marks each. Unit IV having the sessional work/field work(section) could also be a part of the theory paper.

Internship/field work Unit IV having the components/activities of the internship are to be to be developed in the form of the Reflective Journal. All the activities under the internship are to be evaluated for credits and hence all the activities are to be showcased by the trainee and are to be fully recorded with the complete certification of its genuineness.

The Theory paper is to have 60 marks (External). 40 Marks are for the In House activities.

TEACHING OF MATHEMATICS

CONTENTS

LESSON NO. 1 **UNIT-I**

MATHEMATICS STRUCTURE AND KNOWLEDGE

STRUCTURE

1.1 INTRODUCTION

The three most significant developments that differentiate human beings from other species can be named as bipedalism, language development and mathematisation. The ability to think mathematically has resulted in the progress and prosperity of our society. Keeping in mind the importance of mathematics, National Curriculum Framework (2005) has posited that developing children's abilities for mathematisation is the main goal of mathematics education. However, the students even after doing graduation in mathematics are not able to tell what is mathematics? and What are processes of mathematics? It may be interesting for all the students to understand the nature and processes of subject they are following with utmost devotion. The nature of mathematics may also be a topic of interest not only for the teachers of mathematics, but also for all the users of mathematics, like, physicists, geologists, and economists etc., to name a few.

The present lesson may help in understanding nature of mathematics along with processes of mathematics, like, mathematical reasoning, pattern recognition, geometric thinking in relation to Van Hiele model of geometric thought, problem solving in mathematics and creative thinking in mathematics.

1.2 OBJECTIVES

After you study this lesson, you will be able to:

1. Define mathematics.
2. Describe the nature of mathematics.
3. Explain the characteristics of mathematics.
4. Explain the processes of mathematics, like, mathematical reasoning, pattern recognition, geometric thinking in relation to Van Hiele model of geometric thought, problem solving in mathematics and creative thinking in mathematics.

1.3 MEANING, NATURE AND CHARACTERISTICS OF MATHEMATICS

1.3.1 What is Mathematics?

Mathematics is a subject used by all, mastered by some, and cultivated by few has been derived from the Ancient Greek word máthēma (μάθημα). In the modern times the meaning of máthēma can be explained as science, study, learning. Mathematics is a singular noun abbreviated as math or maths. The abbreviation math is used in the U.S. and Canada, while maths is preferred in the most other countries of the world. Although there is no commonly accepted definition, mathematics can be defined as follows:

Oxford Advanced Learner's Dictionary - The science of numbers, quantity, and space.

DK illustrated Oxford Dictionary – Mathematics is the abstract science of numbers, quantity, and space studied in its own right (pure mathematics), or applied to other disciplines such as physics, engineering, etc, (applied mathematics).

Merriam Webster Dictionary - the science of numbers and their operations, interrelations, combinations, generalizations, and abstractions and of space configurations and their structure, measurement, transformations, and generalizations

The Lexicon Webster's Dictionary of the English Language – Mathematics is the science of expressing and studying the relationships between quantities and magnitudes as represented by numbers and symbols.

Benjamin Peirce (1870) – Mathematics is the science that draws necessary conclusions.

Courant and Robbins (1941) - Mathematics is nothing but a system of conclusions drawn from definitions and postulates that must be consistent but otherwise may be created by the free will of the mathematicians.

Davis and Hersh (1986) – Mathematics is similar to an ideology, a religion, or an art form; it deals with human meanings, and is intelligible only within the context of culture.

The National Policy on Education (1992) - Mathematics should be visualised as the vehicle to train a child to think, reason, analyse and to articulate logically.

Courant, Robbins, and Stewart (1996) - Mathematics as an expression of the human mind reflects the active will, the contemplative reason, and the desire for aesthetic perfection. Its basic elements are logic and intuition, analysis and construction, generality and individuality.

Orlicz (2001) - Mathematics is a free flow of thoughts and concepts which a mathematician, in the same way as a musician does with the tones of music and a poet with words, puts together into theorems and theories.

From the above definitions, we can conclude that mathematics is a science (systematic/step-wise way of gathering knowledge) of quantity (amount, size, portion) and space (time, interval, distance). It is a science of abstract relationships that deals with questions involving size, portion, area, time, interval, and distance. It is also a science of calculations involving the use of numbers. Mathematics is numerical part of one's life and deals with logic, analysis and construction. Thus, mathematics is a systematic, organized and exact branch of science that uses abstract concepts.

Check Your Progress

Notes :

(a) write your answers in the space given below.

(b) Compare your answers with those given at the end of the unit.

1. Write definition of mathematics given by the National Policy on Education (1992)

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2. Mathematics is

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1.3.2 Nature and Characteristics of Mathematics

Mathematics is such a vast subject that it presents different faces to different people. So nature of mathematics is a bit tentative. Although, we can get some idea about the nature of mathematics from the definitions of mathematics, the nature of mathematics can be explained from its characteristics given below:

i. Mathematics is Ever Growing

For a layman or a person not belonging to the field of mathematics, mathematics may appear to be static and rigid. But, for mathematicians, mathematics is dynamic and ever growing subject. Mathematics was developed about 5,000 years ago by the Sumerians but every day new findings are coming out that makes mathematics vast and enriched.

ii. Mathematics is Abstract

The nature of mathematics is abstract. The mathematical abstractions are derived either from concrete or from other abstractions. The mathematical symbols of four fundamental mathematical operations (+, ?, ×, ÷), algebra, in which letters and symbols are used to represent numbers and quantities, and geometry that came in to existence as society need to solve problems dealing with distances and area in the real world have been abstracted from real life situations

iii. Mathematics is a Language

Mathematics is a language with well-defined rules. Mathematics as a language has own signs and symbols that cut short the lengthy statements and expressions. Mathematics is a language of science. Mathematics as a language provides exactness and perfection to all sciences. Due to this nature of mathematics, According to Lindsay pointed out that mathematics is the language of physical sciences and certainly no more marvelous language was ever created by the mind of man. Also, mathematics is the language of universe as Galileo Galilei said that mathematics is the language in which God has written the universe.

iv. Mathematics is a Method

Mathematics is a method developed by human mind to simplify the problem of day-to-day life. A problem converted in to mathematical terms can easily be understood and solved. Arithmetic and algebra were discovered to solve the problems of counting and business; trigonometry and geometry were invented for distance and area measurements; calculus arose for physics problems; and statistics helped in getting objective conclusions in various fields.

v. Mathematics is the Science of Logical Reasoning

According to Russel and Whitehead, mathematics is logic. Mathematics starts and ends with logic. Mathematics is the science logical reasoning, wherein results are obtained through the process of logical reasoning. In this relation, the words of Locke are worth mentioning that mathematics is a way to settle in mind the habit of reasoning.

vi. Mathematics is a Science of Inductive and Deductive Reasoning

Mathematics is also a Science of Inductive and Deductive Reasoning. Inductive reasoning means when a particular property is true in a sufficient number of cases, then we can conclude that it will be true in all similar cases. Due to this nature of mathematics, mathematics is considered as inductive in the making. Also, mathematics is a science of deductive reasoning as A.N. Whitehead argued that mathematics in its widest sense is the development of all types of deductive reasoning. The deductive reasoning in mathematics is based on axioms, postulates, undefined terms and definitions.

vii. Mathematics is an Exact and Precise Science

Mathematics is an exact discipline. Mathematical interpretations are clear and accurate; free from ambiguity and conflicting interpretations. The persons belonging to literature and art may have different interpretations for same set of work but this is not the case in mathematics. There is no scope of vagueness and double interpretation in the field of mathematics. For example, a 'point' for all the students of mathematics is a location in space and is represented by a dot. There is no ambiguousness and hidden meaning.

viii. Mathematics is Pure as well as Applied

Mathematics is of two types – Pure and Applied mathematics. Pure mathematics deals with the theories and principles without regard to any application thereof. The abstraction of pure mathematics gives no regard to concrete applications. Albert Einstein said that pure mathematics is, in its way, the poetry of logical ideas. On the other hand, applied mathematics is practical side of mathematics. The researches in the fields of engineering, biology, or social sciences owe much to mathematics. However, mathematicians believe that pure mathematics study sooner or later finds a practical application. In this regard, Nikolai Lobachevsky mentioned that there is no branch of mathematics, however abstract, which may not some day be applied to phenomena of the real world. Moreover, both pure and applied mathematics have their own importance as Edward Frenkel said that it was as though applied mathematics was my spouse, and pure mathematics was my secret lover.

ix. Mathematics is Library

Mathematics is a kind of library used by scientists, engineers, doctors, astronomers, etc. to obtain the solutions of their problems. Reformulation of problems in mathematical terms is necessary most of the times, to get best possible solution.

x. Mathematics is Beautiful

Elegance, beauty and aesthetic charm are internal to mathematical structures. G. H. Hardy has argued that beauty is the first test: there is no permanent place in the world for ugly mathematics. Likewise, Bertrand Russell said that mathematics, rightly viewed, possesses not only truth, but supreme beauty. The beauty of mathematics is evident in patterns, symmetry and elegant proofs. The charm we get on finding right answer to a mathematics problem is hard to express in words. The 'Perspective' used in visual arts has been mathematically based. Also, 'Fibonacci Numbers' provide us amazing sequences; and importance of 'Golden Ratio' has been recognized by painters, sculptors long ago.

xi. Mathematics Believes in Generalizations

Mathematics is not concerned with particular cases. It believes in reaching to generalized truths. For example – geometry moves from two dimensional to infinity of dimensions; number system starts from natural numbers and reaches at complex numbers.

xii. Mathematics is Economical

All mathematical structures and solutions are economical in nature. There is no place of ornamental language in mathematics. No superfluous word is used in mathematics. And, sometimes even necessary words are replaced by signs.

xiii. Mathematics is Creative

R.C. Buck stated that creativity is the heart and soul of mathematics. Mathematics requires creativity. Mathematics is creative enterprise from beginning to end. Ernest (1991) remarked that human mathematical activity is fundamental in the creation of new knowledge and that both mathematical truths and the existence of mathematical objects must be established by constructive methods. Creativity in mathematics is a tool responsible for progression and development of mathematics.

Conclusion

If someone says that mathematics is only what is discussed above, then he/she must be making a blunder. Mathematics is culture specific. We have unlimited possibilities of culture and so is the case with mathematics. Moreover, mathematics is so vast that putting a wire around it is not possible. It is very much possible that new inventions unfold new dimensions of mathematics.

Check Your Progress

Notes :

(a) write your answers in the space given below.

(b) Compare your answers with those given at the end of the unit.

3. What is Pure mathematics?

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4. What is Golden Ratio?

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5. Write five examples of circle from your daily life.

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1.4 PROCESSES OF MATHEMATICS

The processes of mathematics are the ways of engaging in mathematics. The processes of mathematics facilitate learning and using mathematics. The processes of mathematics mean being actively involved in mathematical activities. The NCTM Standards pointed out that "doing mathematics" means engaging in mathematical processes. Also, National Council of Educational Research and Training (NCERT) (2006) emphasized that giving importance to these processes constitutes the difference between doing mathematics and swallowing mathematics, between mathematisation of thinking and memorising formulas, between trivial mathematics and important mathematics, between working towards the narrow aims and addressing the higher aims. Some important mathematics processes are explained in the following captions.

1.4.1 Mathematical Reasoning

Students make sense of mathematics through the process of mathematical reasoning. Without reasoning mathematics learning becomes rote – difficult to understand.

Mathematical reasoning makes mathematics easy to understand. Once a mathematics concept is understood, the students may be able to apply that concept in problem solving and understanding further concepts. Moreover, mathematical reasoning can help in sustaining curiosity in the students.

Michael T. Battista stated that reasoning is the process of manipulating and analyzing objects, representations, diagrams, symbols, or statements to draw conclusions based on evidence or assumptions. Developing student' ability to reason should be the basic aim of mathematics teacher. A good teacher of mathematics needs to premise his/her teaching on mathematical reasoning. He/she must ask 'why this…..'; 'why do you think so'; 'What is your argument'; 'Explain your answer', 'How you got this result'; and 'How this has come up'. According to NCERT (2006) the aim of process of reasoning should be to develop arguments, evaluate arguments, make and investigate conjectures, and understand that there are various methods of reasoning. Russell (1999) identifies four important points about active mathematics reasoning in school classrooms:

(1) reasoning is about making generalizations,

(2) reasoning leads to a web of generalizations,

(3) reasoning leads to mathematical memory built on relationships, and

(4) learning through reasoning requires making mistakes and learning from them.

For developing students' mathematical reasoning, a teacher needs to focus on three main components to the reasoning process - conjecturing, generalizing and justifying. Also, Mata-Pereira and Ponte (2017) stated that mathematical reasoning processes include formulating questions and solving strategies, formulating and testing generalizations and other conjectures, and justifying them.

Thus, the process of mathematical reasoning can be effective in helping students make sense of mathematics they are learning. However, the key is willingness of the teacher to provide variety of challenging tasks and attitude of cultivating curiosity.

1.4.2 Pattern Recognition

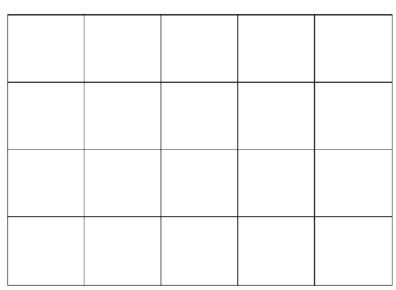
Pattern recognition is an important ability of human beings. Advancement in mathematics is also related students' ability to recognize underlying patterns. The patterns may not be visual at once, but with mind's ability one can see regularity in the items under consideration.

J.N. Kapur (1969) pointed out that all scientific activity consists in perceiving of patterns and structures in nature, society and industry. One of the most important roles of school mathematics is accordingly to strengthen this perception of patterns and structures in the domains where their perception is simplest, where these occur almost in the pure form and where their study can be really fascinating to the children *viz.* in the domains of numbers and space. After a pattern has been perceived through intuition, it has to be established rigorously later.

NCTM Standard 2 (1998) advocated that mathematics instructional programs should include attention to patterns, functions, symbols, and models so that all students understand various types of patterns and functional relationships; use symbolic forms to represent and analyze mathematical situations and structures; and use mathematical models and analyze change in both real and abstract contexts.

The recognition of patterns not only helps students in understanding mathematics, it also encourages natural curiosity of students. The teachers need encourage students for seeing pattern in various aspects of daily life as well as mathematics. The illustration of pattern may also be fun-filled activity for the students. Some examples of pattern recognition are given in the following:

Example 1: What is area of this rectangle? Find area by counting all the squares. Each square is of area 1 cm^2. Explain how you got your answer.



A student may respond to this problem either by counting all the 20 squares or by recognizing the pattern that there are four row and each row has 5 squares so there are 20 squares.

Example 2 :

$9 \times 1 = 9 = 9$

$9 \times 2 = 18 = 1 + 8 = 9$

$9 \times 3 = 27 = 2 + 7 = 9$

$9 \times 4 = 36 = 3 + 6 = 9$

$9 \times 5 = 45 = 4 + 5 = 9$

$9 \times 6 = 54 = 2 + 7 = 9$

$9 \times 7 = 63 = 6 + 3 = 9$

$9 \times 8 = 72 = 7 + 2 = 9$

$9 \times 9 = 81 = 8 + 1 = 9$

$9 \times 10 = 90 = 9 + 0 = 9$

Example 3:

$37 \times 3 = 111$

$37 \times 6 = 222$

$37 \times 9 = 333$

$37 \times 12 = 444$

1.4.3 Algebraic Thinking

Although algebraic thinking has existed for over 3000 years, many students still have difficulties in algebra. Algebra is different from arithmetic in relation to abstract character.

Lins (1992) - To think algebraically is,

(i) To think ARITHMETICALLY that is modeling in numbers, and

(ii) To think INTERNALLY, which means reference only to the operations and equality relations, and

(iii) To think ANALYTICALLY, which means unknown has to be treated as known.

Kieran (1996) defined algebraic thinking as the use of a variety of representations in order to handle quantitative situations in a relational way.

Driscoll (1999) – A facility with algebraic thinking includes being able to think about 'functions' and how they work, and to think about the impact that a system's 'structure' has on calculations.

Cathy Seeley (2004) - Algebraic thinking includes recognizing and analyzing patterns, studying and representing relationships, making generalizations, and analyzing how things change.

NCTM (2014) – The three main themes of algebraic thinking are: thinking relationally about equality, thinking rule-wise in pattern generalization, and thinking representationally about the relations in problem situations.

Mustaffa, Ismail, Said, and Tasir (2017) - Algebraic thinking is about the ability of students to analyze, make generalizations, solve problems, predict, justify, and prove as well as model some situations.

Thus, algebraic thinking is facility, to think in mathematical terms as compared to situational ones; to understand patterns and functions; and thinking relationally about equality.

In the Principles and Standards for School Mathematics (NCTM, 2000), four goals are listed related to the algebra strand:

Goal 1—Understand patterns, relations, and functions;

Goal 2—Represent and analyze mathematical situations and structures using algebraic symbols;

Goal 3—Use mathematical models to represent and understand quantitative relationships; and

Goal 4—Analyze change in various contexts.

In order to promote algebraic thinking, a teacher of mathematics needs to incorporate following approaches in his/her teaching:

Working backward

A teacher needs to provide such situations and problems to the students that they are able to work backward. At elementary level problems like 2 + ______ = 7 and _____ + _____ = 7 can be asked, while at secondary level students can be asked to find an equation whose solutions are 4 and - 5. This working backward approach will provide training in reversibility, which is an important aspect of algebraic thinking.

Making Generalizations

The teachers need to motivate students to generalize the patterns recognized. Generalization means applying a given argument in a broader context (Harel & Tall, 1991). Expressing generality is one of the roots of, and routes into, algebra (Mason, 1996). Studnets must be able to tell, "What's the rule for this pattern?" The students can be asked to write 100th stage in the following example:

□ □□	□□ □□□	□□□ □□□□	□□□□ □□□□□
Stage 1	Stage 2	Stage 3	Stage 4

Abstraction of Concrete

Promoting students' abstraction of concrete/specific cases is essential for developing algebraic thinking. Abstraction is possible when a learner is able to perceive analogies and relationships. This can be done by asking questions involving why and how, by asking more similar examples, by asking to evaluate answer and draw inferences.

Thus, algebraic thinking may be promoted when students are asked, to reason about patterns, to see the general in the particular, to reverse operations and to think about mathematical relations.

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1.4.4 Geometric Thinking (Van Hiele Model of Geometric Thinking)

Geometry has been traced from the ancient Greek: geo- “earth”, metron- “measurement”. Geometry is used in our everyday life, but the methods adopted for teaching geometry are far from satisfactory. They are rigid and traditional. New methods are hardly tried by the teachers. The Van Hiele Model given in 1957 by Dina van Hiele-Geldof and Pierre M. Van Hiele is an effective method for promoting geometric thinking among students.

According to Van Hiele, geometric thought develops in sequence of five levels. These levels ranged from the lowest to the highest. The NCTM monograph on Van Hiele Model of Geometric Thinking describes Van Hiele’s Model as follows:

- **Level 0 (Visualization)**

  The student reasons about basic geometric concepts, such as simple shapes, primarily by means of visual consideration of the concept as a whole without explicit regard to properties of its components. The student identifies, names, compares and operates on geometric figures (e.g., triangles, angles, intersecting or parallel lines) according to their appearance. For example, Level 0: Student measures angles of a parallelogram

- **Level 1 (Analysis)**

  The student reasons about geometric concepts by means of an informal analysis of component parts and attributes. Necessary properties of the concept are established. The student analyzes figures in terms of their components and discovers properties/rules of a class of shapes empirically.

For example, the student discovers that opposite angles of parallelograms are equal by colouring in equal angles in a grid of parallelograms.

- **Level 2 (Abstraction)**

  The student logically orders the properties of concepts, forms abstract definitions, and can distinguish between the necessity and sufficiency of set of properties in determining a concept. The student logically interrelates previously discovered properties/rules by giving or following informal arguments. For example, Student gives an informal argument why opposite angles are equal using known principles (e.g., saw or ladder); students explains that why two acute angles in any right triangle add up to 90^0?

- **Level 3 (Deduction)**

  The student reasons formally within the context of a mathematical system, complete with undefined terms, axioms, an underlying logical system, definitions and theorems. The student proves theorems deductively and establishes interrelationship among networks of theorems. For example, students give examples of axioms, postulates, and theorems in Euclidean plane geometry and describe how they are related.

- **Level 4 (Rigor)**

  The student can compare systems based on different axioms and can study various geometries in the absence of concrete models. The student establishes theorems in different postulational systems and analyzes/compares these systems. For example, the students invent generalized methods for solving classes of problems.

According to Pierre Van Hiele (1959/1984), progress from one level to the next involves five phases: information, guided orientation, explicitation, free orientation, and integration. The phases, which lead to a higher level of thought, are described as follows with examples given for transition from level 0 to level 1. (Fuys, Geddes, & Tischler, 1988)

Information: The student acquainted with the working domain. (e.g., examines examples and non-examples).

Guided orientation: The student does tasks involving different relations of the network that is to be formed (e.g., folding, measuring, and looking for symmetry).

Explicitation: The student become conscious of the relations, try to express them in words and learns technical language which accompanies the subject matter (e.g., expresses idea about the property of figures).

Free orientation: The students learn by doing more complex tasks to find his/ her own way in the network of relations (e.g., knowing properties if one kind of shape, investigate these properties for a new shape, such as kites).

Integration: The student summaries all that he/she learned about the subject, then reflect his/her actions and obtains and overview of newly formed network of relations now available (e.g., properties of figures are summarized).

Despite so much research and benefits, the Van Hiele model has not find any space in the mathematics curriculum prescribed by various school boards. If properly introduced in the curriculum, the Van Hiele Model can help in enhancing geometric thinking.

1.4.5 Problem Solving in Mathematics

Problem solving, as level 8, is the highest type of learning within Gagné's hierachy. Problem solving in mathematics can help students become more effective in understanding mathematics and solving mathematics problems. The problem-solving based learning environments enable students to develop deep mathematics knowledge and give them the opportunity of building their own mathematics (Schoenfeld, 1992).

A problem consists of a given state (i.e., a description of the current situation), a goal state (i.e., a description of the desired situation), and a set of operators (i.e., rules or procedures for moving from one state to another) (Mayer, 1985). Problem solving in mathematics can help in attaining the desired goal.

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Problem solving in mathematics can be defined as follows:

> Polya (1945) - Problem solving is finding a way out of a difficulty, a way around an obstacle, attaining an aim that was not immediately attainable.
>
> Newell and Simon (1972) - Problem solving is a search for a path between the given and goal state of a problem.
>
> Schoenfeld (1992) - Problem solving in mathematics refers to the process wherein students encounter a problem – a question for which they have no immediately apparent resolution, nor algorithm that they can directly apply to get an answer.
>
> Mayer and Wittrock (2006) - Problem solving is a cognitive processing directed at achieving a goal when no solution method is obvious to the problem solver.
>
> Cai and Lester (2010) – Problem solving refers to mathematical tasks that have the potential to provide intellectual challenges for enhancing students' mathematical understanding and development.

Thus, problem solving in mathematics not only helps in arriving at a correct solution, it stimulates mathematics ability of students.

The problem solving in mathematics is a systematic procedure to reach the desired goal. Various mathematics educators have given step for problem solving in mathematics. However, Polya (1945), a famous mathematician, has developed a unique and perhaps the most appropriate theory of problem solving in mathematics. He suggested a broader scheme of four stages in mathematical problem solving. These four stages are as follows:

1. UNDERSTANDING THE PROBLEM - You have to understand the problem.

   - What is unknown? What are the data? What is the condition?
   - Is it possible to satisfy the condition? Is the condition sufficient to determine the unknown? Or is it insufficient? Or redundant?
   - Draw a figure. Introduce suitable notation.
   - Separate the various parts of condition. Can you write them down?

2. DEVISING A PLAN - Find the connection between the data and the unknown

   - Have you seen it before? Or have you seen the same problem in a slightly different form?
   - Do you know a related problem? Do you know a theorem that could be useful?
   - Look at the unknown! And try to think of a familiar problem having the same or a similar unknown.
   - Here is a problem related to yours and solved before. Could you use it? Could you use its result? Could you use its method? Should you introduce some auxiliary element in order to make use possible?
   - Could you restate the problem? Could you restate it still differently?
   - If you cannot solve the proposed problem try to solve first some related problem.

3. CARRYING OUT THE PLAN – Carry Out Your Plan

    - Carrying out your plan of the solution, check each step. Can you see clearly that the step is correct? Can you prove that it is correct?

4. LOOKING BACK – Examine the Solution Obtained

    - Can you check the result? Can you check the argument?
    - Can you derive the result differently? Can you see it at a glance?
    - Can you use the result, or the method, for some other problem?

Advantages of Problem Solving in Mathematics

i. Problem solving in mathematics helps students in becoming independent in solving problems of real life.

ii. The problem solving in mathematics helps in establishing better teacher-taught relationships.

iii. The problem solving in mathematics develops patience and persistence in students.

iv. It also develops the quality of curiosity among students.

v. The problem solving in mathematics helps in removing ills of some of the school education problems, like, indiscipline, learning by rote, lack of interest, etc.

vi. The problem solving in mathematics helps in achieving the aim of heuristic learning.

vii. The problem solving in mathematics helps student in becoming self-reliant and self-confident in mathematics.

viii. The learning done with problem solving in mathematics is durable in nature.

ix. The problem solving in mathematics is according to the psychological principles of teaching-learning. It is child-centered and problem oriented.

x. Problem solving in mathematics helps student in solving problems in a systematic manner.

Disadvantages of Problem Solving in Mathematics

i. There is lack of teachers trained in problem solving in mathematics.

ii. The problem solving in mathematics is quite challenging for average and below average students.

iii. The problem solving in mathematics requires scientific enquiry and reflective thinking, which is not a cup of tea for each and every teacher as well as student.

iv. The syllabus may not be covered in time with the use of this problem solving in mathematics.

v. The problem solving in mathematics cannot be used effectively in Indian scenario because of overcrowded classes, paucity of funds, lack of library and laboratory facilities, too much consideration to summative evaluation, lack of willingness of teachers, and lack of cooperation of parents.

1.4.6 Creative Thinking in Mathematics

Another important process of mathematics is creativity in mathematics. According to National Curriculum Framework (2005), the development of self-esteem and ethics, and the need to cultivate children's creativity, must receive primacy. Creativity in all the subjects need to be encouraged and facilitated. And, mathematics is such a subject where creative thinking can be easily identified. Vale and Barbosa (2015) remarked that creativity is a topic that should be part of the mathematics programs at all educational levels, but it is still much neglected in mathematics classes, because teachers are unaware and/or have not yet perceived its importance. Creative thinking in mathematics has been conceptualized by various mathematics educators as follows:

Kapur (1977) - The purest forms of creative effort is in mathematics; it does not greatly depend on availability of equipment, or on complexity of social phenomenon.

Hadamard (1945) identified the ability to find key research questions as an indicator of exceptional talent in the domain of mathematics.

Haylock (1987) - Mathematical creativity is the ability to break from mental sets, overcome fixations and production of many, varied and original responses in open-ended situations in Mathematics.

Ernest (1991) - Mathematics is social institution, resulting from human problem posing and solving.

Sharma (2013) - The common characteristics of creative thinking in mathematics are:

(a) the ability to overcome fixations in mathematical situations,

(b) the ability to formulate mathematical problems, and

(c) ability to solve a mathematical problem with multiple solutions.

As a result, mathematical creativity is an ability to overcome fixation as well as conceptualizing, proposing, and even testing unusual solutions of problem(s) of Mathematics.

In order to develop mathematical creativity, the classroom activities and curricular materials are needed to be reorganized. Sharma (2009) has given Strategy for Fostering Mathematical Creativity. The Strategy for Fostering Mathematical Creativity has three phases. Phase I was warm up the class with divergent mathematical problem. While, second and third phases were cooperative confrontation and independent thought respectively. The task of the first phase is to sensitize the students about the unlimited number of responses that a mathematical problem can have. In this phase, a mathematical problem demanding divergent answers is posed by the teacher. This problem is to be solved by the teacher with the active participation of the students. Students need to be encouraged and prompted to think differently. Efforts are made to get at least one response from each student.

The second phase of the Strategy for Fostering Mathematical Creativity is aimed to bring into play the synergy for finding various solutions of a divergent mathematics problem. This involves posing of divergent mathematics problem by the teacher and solution of that by the students in a group. The students are grouped and each group consist of 4-5 students. During the process teacher motivates the groups by saying that let us see which group comes out with maximum and different answers. For the responses feedback need to be provided.

In phase three each student will try to pose a problem similar to the one solved in the second phase. Students will then solve the problems they had posed without the help from the teacher or the peer(s). At the end teacher will write the most original responses of the students on the black-board. The role of the teacher is supportive and motivating.

Creativity in mathematics is an important process of mathematics. Apart from helping students to understand mathematics, creativity in mathematics is necessary for progress of mathematics itself. So, every effort should be made to encourage students' creative thinking in mathematics.

Check Your Progress

Notes :

(a) Write your answers in the space given below.

(b) Compare your answers with those given at the end of the unit.

6. What do you mean by processes of mathematics?

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7. What are important processes of mathematics?

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8. Write approaches for promoting algebraic thinking.

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9. Explain levels of Van Hiele Model of Geometric Thinking in brief.

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10. What are Polya's steps for problem solving in mathematics?

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11. What is creative thinking in mathematics?

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1.5 LET US SUM UP

Mathematics is a science of quantity and space. Although mathematics is such a vast subject that we cannot put a boundary around it, the nature of mathematics can be comprehended from its characteristics, like, mathematics is ever growing; mathematics is abstract; mathematics is a language; mathematics is a method; mathematics is the science of logical reasoning; mathematics is a science of inductive and deductive reasoning; mathematics is an exact and precise science; mathematics is pure as well as applied; mathematics is library; mathematics is beautiful; mathematics believes in generalizations; mathematics is economical; and mathematics is creative. The lesson also provides understanding of important processes in mathematics that can help in making students active and proficient in mathematics learning.

1.6 LESSON-END EXERCISE

1. Write meaning and nature of mathematics.
2. What is mathematics? Discuss important characteristics of mathematics?
3. What do you mean by processes of mathematics? Describe algebraic thinking and pattern recognition processes of mathematics.
4. Explain briefly the processes of mathematics. How as a teacher you can encourage these processes of mathematics? Give examples.

1.7 SUGGESTED FURTHER READINGS

Fuys, D., Geddes, D., & Tischler, R. (1988). The van Hiele model of thinking in geometry among adolescents [Monograph]. *Journal for Research in Mathematics Education*, 3, 1-196.

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NCTM. (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: Author.

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Polya, G. (1945). *How to solve it; a new aspect of mathematical method*. Princeton, NJ, US: Princeton University Press.

Sharma, Y. (2014). The Effects of Strategy and Mathematics Anxiety on Mathematical Creativity of School Students. *Mathematics Education*, 9(1), 25-37

1.8 ANSWERS TO CHECK YOUR PROGRESS

1. Mathematics should be visualised as the vehicle to train a child to think, reason, analyse and to articulate logically.
2. a science of abstract relationships that deals with questions involving size, portion, area, time, interval, and distance.
3. Pure mathematics is part of mathematical activity that is done without explicit or immediate consideration of direct application.
4. The golden ratio, is known as the divine proportion, golden mean, or golden section. The number $\frac{1+\sqrt{5}}{2}$ is called golden ratio and is approximately equal to 1.618.
5. Human face, sun, full moon, chapatti (Indian bread), coin.
6. The processes of mathematics are the capacities that students need to acquire for understanding and doing mathematics.
7. Algebraic Thinking, Mathematical Reasoning, Pattern Recognition, Geometric Thinking, Problem Solving, Creative Thinking in Mathematics, Reasoning and Proof, Communication, Connections, and Representation are important processes of mathematics.
8. Working backward, Making Generalizations, and Abstraction of Concrete.

9. The levels of Van Hiele's Model of Geometric Thinking are:

   Level 0 (Visualization) - The student identifies, names, compares and operates on geometric figures (e.g., triangles, angles, intersecting or parallel lines) according to their appearance.

   Level 1 (Analysis) - The student analyzes figures in terms of their components and discovers properties/rules of a class of shapes empirically.

   Level 2 (Abstraction) - The student logically interrelates previously discovered properties/rules by giving or following informal arguments.

   Level 3 (Deduction) - The student reasons formally within the context of a mathematical system, complete with undefined terms, axioms, an underlying logical system, definitions and theorems.

   Level 4 (Rigor) - The student establishes theorems in different postulational systems and analyzes/compares these systems.

10. These four stages are as follows:

    UNDERSTANDING THE PROBLEM - You have to understand the problem.

    DEVISING A PLAN - Find the connection between the data and the unknown

    CARRYING OUT THE PLAN – Carry Out Your Plan

    LOOKING BACK – Examine the Solution Obtained

11. Mathematical creativity is an ability to overcome fixation as well as conceptualizing, proposing, and even testing unusual solutions of problem(s) of Mathematics.

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LESSON NO. 2 **UNIT-I**

STRUCTURE OF MATHEMATICS

STRUCTURE

2.1 INTRODUCTION

Mathematics is an abstract science that deals with mathematical structures. By the structure of mathematics we mean a set with specified functions and relations satisfying specified axioms. The structure of mathematics gives importance upon making combinations under some specific rules. The structure of mathematics has been organized upon other mathematical substructures. These three basic structures are called algebraic structure, topological structure, and order structure. NCTM Curriculum standards for grades 9-12 has emphasized the study of structures in mathematics so that all students can—

a. compare and contrast the real number system and its various subsystems with regard to their structural characteristics;

b. understand the logic of algebraic procedures;

c. appreciate that seemingly different mathematical systems may be essentially the same;

and so that, in addition, college-intending students can—

d. develop the complex number system and demonstrate facility with its operations;

e. prove elementary theorems within various mathematical structures, such as groups and fields;

f. develop an understanding of the nature and purpose of axiomatic systems.

The present lesson may help in understanding important structures in mathematics, namely, Euclidean geometry, terms (undefined and defined terms); axioms, postulates and theorems, and validation process of mathematical statements.

2.2 OBJECTIVES

After you study this lesson, you will be able to:

1. Explain the meaning of structure of mathematics.
2. Explain the Euclidean geometry - terms (undefined and defined terms); axioms, postulates and theorems
3. Describe validation process of mathematical statements.

2.3 STRUCTURE OF MATHEMATICS

2.3.1 Euclidean Geometry

Euclidean Geometry got its name after famous mathematician Euclid (born c. 300 BCE, Alexandria, Egypt). Our modern geometry has been rooted on book of Euclid – Elements. The Euclidean geometry has been based upon the axiomatic system. The Euclid's first proposition says that an equilateral triangle can be constructed on any given base. Euclidean geometry provides us the ideas of mathematical proof and the logical development of a subject. Euclidean geometry is about proving mathematical results. The students are expected to prove geometrical truths with the help of axioms, postulates and previously proved truths.

The Euclidean geometry has a proper sequence - definitions, postulates and axioms, the statements of the theorems and their proofs.

Terms (Defined and Undefined)

In Euclidean geometry, Point, Line, and Plane are all undefined terms. All other terms in geometry can be defined from these three undefined terms. Although, point, line and plane are undefined terms, Euclid described them as:

1. A *point* is that of which there is no part.
2. A *line* is a length without breadth.
3. The extremities of a line are points.
4. A *straight-line* is (any) one which lies evenly with points on itself.
5. A *surface* is that which has length and breadth only.
6. The ends of a surface are lines.
7. A *plane surface* is (any) one which lies evenly with the straight-lines on itself.
8. A *plane angle* is the inclination of the lines to one another, when two lines in a plane meet one another, and are not lying in a straight-line.
9. When the lines containing the angle are straight then the angle is called *rectilinear*.
10. When a straight-line stood upon (another) straight-line makes adjacent angles (which are) equal to one another, each of the equal angles is a *right-angle*, and the former straight-line is called a perpendicular to that upon which it stands.

11. An *obtuse angle* is one greater than a right-angle.

12. *Quadrilateral* figures: a *square* is that which is right-angled and equilateral, a *rectangle* that which is right-angled but not equilateral, a *rhombus* that which is equilateral but not right-angled, and a *rhomboid* that having opposite sides and angles equal to one another which is neither right-angled nor equilateral. And let quadrilateral figures besides these be called *trapezia*.

13. *Parallel lines* are straight-lines which, being in the same plane, and being produced to infinity in each direction, meet with one another in neither (of these directions).

Common Notions (Axioms)

Axioms are statements or principles that are generally accepted to be true, but need not be so. Axioms are mathematical facts that are accepted without proof. In order to prove a theorem we need some starting points or some building blocks, axioms are those building blocks. They are used all over in the mathematics. The axioms of Euclidean geometry are as follows:

1. Things which are equal to the same thing are also equal to one another.
2. If equals be added to equals, the wholes are equal.
3. If equals be subtracted from equals, the remainders are equal.
4. Things which coincide with one another are equal to one another.
5. The whole is greater than the part.

Postulates

Axioms and postulates are interchangeable terms. However, postulates aim to capture what is special about a particular structure. The postulates deal with specificity.

1. To draw a straight line from any point to any point.
2. To produce a finite straight line continuously in a straight line.
3. To describe a circle with any center and distance.

4. That all right angles are equal to one another.
5. That, if a straight line falling on two straight lines make the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which are the angles less than the two right

Propositions (Theorems)

The logical consequence of the axioms is called propositions. The propositions are all theorems. The propositions or theorems are derived using the axioms and postulates. The propositions of Euclidean geometry are:

1. On a given finite straight line to construct an equilateral triangle.
2. To place at a given point (as an extremity) a straight line equal to a given straight line.
3. Given two unequal straight lines, to cut off from the greater a straight line equal to the less.
4. If two triangles have the two sides equal to two sides respectively, and have the angles contained by the equal straight lines equal, they will also have the base equal to the base, the triangle will be equal to the triangle, and the remaining angles will be equal to the remaining angles respectively, namely those which the equal sides subtend.
5. In isosceles triangles the angles at the base are equal to one another, and, if the equal straight lines be produced further, the angles under the base will be equal to one another.
6. If in a triangle two angles be equal to one another, the sides which subtend the equal angles will also be equal to one another.
7. Given two straight lines constructed on a straight line (from its extremities) and meeting in a point, there cannot be constructed on the same straight line (from its extremities), and on the same side of it, two other straight lines meeting in another point and equal to the former two respectively, namely each to that which has the same extremity with it.

8. If two triangles have the two sides equal to two sides, respectively, and have also the base equal to the base, they will also have the angles equal which are contained by the equal straight lines.
9. To bisect a given rectilinear angle.
10. To bisect a given finite straight line.
11. To draw a straight line at right angles to a given straight line from a given point on it.
12. To a given infinite straight line, from a given point which is not on it, to draw a perpendicular straight line.
13. If a straight line set up on a straight line make angles, it will make either two right angles or angles equal to two right angles.
14. If with any straight line, and at a point on it, two straight lines not lying on the same side make the adjacent angles equal to two right angles, the two straight lines will be in a straight line with one another.
15. If two straight lines cut one another, they make the vertical angles equal to one another.
16. In any triangle, if one of the sides be produced, the exterior angle is greater than either of the interior and opposite angles.
17. In any triangle two angles taken together in any manner are less than two right angles.
18. In any triangle the greater side subtends the greater angle.
19. In any triangle the greater angle is subtended by the greater side.
20. In any triangle two sides taken together in any manner are greater than the remaining one.
21. If on one of the sides of a triangle, from its extremities, there be constructed two straight lines meeting within the triangle, the straight lines so constructed will be less than the remaining two sides of the triangle, but will contain a greater angle.

22. Out of three straight lines, which are equal to three given straight lines, to construct a triangle: thus it is necessary that two of the straight lines taken together in any manner should be greater than the remaining one.

23. On a given straight line and at a point on it to construct a rectilinear angle equal to a given rectilinear angle.

24. If two triangles have the two sides equal to two sides respectively, but have the one of the angles contained by the equal straight lines greater than the other, they will also have the base greater than the base.

25. If two triangles have the two sides equal to two sides respectively, but have the base greater than the base, they will also have the one of the angles contained by the equal straight lines greater than the other.

26. If two triangles have two angles equal to two angles respectively, and one side equal to one side, namely, either the side adjoining the equal angles, or that opposite one of the equal angles, then the remaining sides equal the remaining sides and the remaining angle equals the remaining angle.

27. If a straight line falling on two straight lines makes the alternate angles equal to one another, then the straight lines are parallel to one another.

28. If a straight line falling on two straight lines makes the exterior angle equal to the interior and opposite angle on the same side, or the sum of the interior angles on the same side equal to two right angles, then the straight lines are parallel to one another.

29. A straight line falling on parallel straight lines makes the alternate angles equal to one another, the exterior angle equal to the interior and opposite angle, and the sum of the interior angles on the same side equal to two right angles.

30. Straight lines parallel to the same straight line are also parallel to one another.

31. To draw a straight line through a given point parallel to a given straight line.

32. In any triangle, if one of the sides is produced, then the exterior angle equals the sum of the two interior and opposite angles, and the sum of the three interior angles of the triangle equals two right angles.

33. Straight lines which join the ends of equal and parallel straight lines in the same directions are themselves equal and parallel.

34. In parallelogrammic areas the opposite sides and angles equal one another, and the diameter bisects the areas.

35. Parallelograms which are on the same base and in the same parallels equal one another.

36. Parallelograms which are on equal bases and in the same parallels equal one another.

37. Triangles which are on the same base and in the same parallels equal one another.

38. Triangles which are on equal bases and in the same parallels equal one another.

39. Equal triangles which are on the same base and on the same side are also in the same parallels.

40. Equal triangles which are on equal bases and on the same side are also in the same parallels.

41. If a parallelogram has the same base with a triangle and is in the same parallels, then the parallelogram is double the triangle.

42. To construct a parallelogram equal to a given triangle in a given rectilinear angle.

43. In any parallelogram the complements of the parallelograms about the diameter equal one another.

44. To a given straight line in a given rectilinear angle, to apply a parallelogram equal to a given triangle.

45. To construct a parallelogram equal to a given rectilinear figure in a given rectilinear angle.

46. To describe a square on a given straight line.

47. In right-angled triangles the square on the side opposite the right angle equals the sum of the squares on the sides containing the right angle.

48. If in a triangle the square on one of the sides equals the sum of the squares on the remaining two sides of the triangle, then the angle contained by the remaining two sides of the triangle is right.

Critique of Euclidean's Geometry

1. The definitions are not sufficient to fully describe the terms being defined.
2. The axioms are not stated precisely. Some of the axioms are not true.
3. The Euclidean geometry tries to define everything. However, the modern geometry accepts some terms as undefined terms.
4. In the present times, the postulates have been found to be both incomplete and superabundant.
5. Euclidean's Geometry is largely 2-Dimensional.

However, the in Euclidean geometry on need not to cram independent facts but from a small set of axioms, one could reconstruct the entire collection of geometric truths.

Check Your Progress

Notes :

(a) Write your answers in the space given below.

(b) Compare your answers with those given at the end of the unit.

1. Write first two Axioms of Euclidean geometry.

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2. Euclidean geometry is about 3D space. True/False

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3. Euclidean geometry is valid only for curved surfaces. True/False

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2.3.2 Validation Process of Mathematical Statements

Validation process of mathematical statements is essential for students mathematics learning. Validation process of mathematical statements asks students to apply through reasoning. Shelden and Shalden (1995) used the term validation to describe the process an individual carries out to determine whether a proof is correct and actually proves the particular theorem it claims to prove.

The validation of mathematical statement requires checking whether a statement is true or not.

According to Selden and Selden (1999), validation can include following:

1. Asking and answering questions,
2. Assenting to claims,
3. Constructing subproofs,
4. Remembering or finding and interpreting other theorems and definitions,
5. Complying with instructions (e.g., to consider or name something), and
6. Conscious (but probably nonverbal) feelings of rightness or wrongness.
7. The production of a new text-a validator-constructed modification of the written argument- that might include additional calculations,
8. Expansions of definitions, or constructions of subproofs. and
9. Toward the end of a validation, in an effort to capture the essence of the argument in a single train of thought, contractions of the argument might be undertaken.

The mental process when validating proofs can include for example asking/answering questions, constructing subproofs or recalling other theorems and definitions (Selden & Selden, 2003).

Validation of mathematical statements is a type of reflection where students are supposed to make claims, draw inferences, ask and answer questions, bring in outside knowledge, construct visual images, or develop subproofs (Selden & Selden, 1999).

NCERT has given a list some general rules for checking whether a statement is true or not. The list of rules is as follows:

Rule 1: If p and q are mathematical statements, then in order to show that the statement *"p and q"* is true, the following steps are followed.

Step-1 Show that the statement p is true.

Step-1 Show that the statement q is true.

Rule 2: Statements with "Or"

If p and q are mathematical statements, then in order to show that the statement *"p or q"* is true, one must consider the following.

Step-1 By assuming that p is false, show q must be true.

Step-1 By assuming that q is false, show p must be true.

Rule 3: Statements with *"If-then"*

In order to prove the statement "if p then q" we need to show that any one of the following case is true.

Case 1 By assuming the p is true, prove that q must be true. (Direct Method)

Case 1 By assuming the q is false, prove that p must be false. (Contrapositive Method)

Rule 4: Statements with *"If and only if"*

In order to prove the statement "p if and only if q" we need to show.

(i) If p is true, then q is true and

(ii) If q is true, then p is true

Some of the methods that can be used in this connection are:

(a) Direct method: Assume p is true and show q is true, i.e., p ? q.

(b) Contrapositive method: Assume ~ q is true and show ~ p is true, i.e., ~ q ? ~ p.

(c) Contradiction method: Assume that p is true and q is false and obtain a contradiction from assumption.

(d) By giving a counter example: To prove the given statement r is false we give a counter example. Consider the following statement.

"r : All prime numbers are odd". Now the statement 'r' is false as 2 is a prime number and it is an even number.

A student of mathematics must be able to validate mathematical statement. For this to happen, effective training should be provided to teachers in teacher training courses. Shelden and Shelden (1995) stated that for effective validation to occur, the students should be encouraged to attend more to possible global/structural errors, for example, proving the converse of the statement.

Check Your Progress

Notes :

(a) Write your answers in the space given below.

(b) Compare your answers with those given at the end of the unit.

4. What is validation?

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5. What are methods for validation of mathematical statements?

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2.4 LET US SUM UP

Euclidean Geometry got its name after famous mathematician Euclid. There are twenty three definitions, five axioms, five postulates and forty eight propositions in Euclidean geometry. In Euclidean geometry, mathematical results are proved with the help of axioms, postulates and previously proved truths. Moreover, the validation of mathematical statement is about checking whether a statement is true or not. Although validation of mathematical statements is explicit in nature, it should be part of mathematics curriculum at various levels.

2.5 LESSON-END EXERCISE

1. Explain the meaning of structure of mathematics.
2. Discuss Euclidean geometry in relation to terms (undefined and defined terms); axioms, postulates and theorems
3. Describe validation process of mathematical statements. Give an example also.

2.6 SUGGESTED FURTHER READINGS

Baralis, G., Malafekas, T., Rappos, E., & Vlamos, P. M. (2000). The instruction of Euclidean geometry with the use of educational software in multilingual classes. *The Teaching of Mathematics*, III(2), 105-113.

Fitzpatrick, R. (2008). Euclid's elements of geometry. http://farside.ph.utexas.edu/Books/Euclid/Elements.pdf

Selden, A., & Selden, J. (1995). Unpacking the logic of mathematical statements. *Educational Studies in Mathematics*, 29, 123-151.

Selden, A., & Selden, J. (1999). *The role of logic in the validation of mathematical proofs*. Cookeville:Tennessee Technological University.

Selden, A., & Selden, J. (2003). Validations of proofs considered as texts: Can Undergraduates tell whether an argument proves a theorem? *Journal for Research in Mathematics Education*, 34(1), 4-36.

2.7 ANSWERS TO CHECK YOUR PROGRESS

1. i. Things which are equal to the same thing are also equal to one another.

   ii. If equals be added to equals, the wholes are equal.

2. FALSE

3. FALSE

4. Validation is the process an individual carries out to determine whether a proof is correct and actually proves the particular theorem it claims to prove.

5. i. Direct method

   ii. Contrapositive method

   iii. Contradiction method

   iv. By giving a counter example

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LESSON NO. 3 **UNIT-I**

MATHEMATICS STRUCTURE AND KNOWLEDGE

STRUCTURE

3.1 INTRODUCTION

3.2 OBJECTIVES

3.3 PEDAGOGIC CONTENT KNOWLEDGE ANALYSIS FOR –FACTS, CONCEPTS, GENERALIZATIONS, AND PROCEDURES

3.4 KNOWLEDGE AND TEACHING OF INTEGERS, RATIONAL NUMBER, REAL NUMBER, POLYNOMIALS, QUADRATIC EQUATIONS, AND CONGRUENCY OF TRIANGLES.

3.5 LET US SUM UP

3.6 LESSON–END EXERCISE

3.7 SUGGESTED FURTHER READINGS

3.8 ANSWERS TO CHECK YOUR PROGESS

3.1 INTRODUCTION

Mathematics teachers are always worried about the learning of their students. They are always concerned with how and in what ways mathematics learning of the students can be improved. The present chapter is devoted to pedagogic content knowledge analysis for – facts, concepts, generalizations, and procedures of mathematics and about knowledge of certain concepts of mathematics. By understanding these, we may be able to teach better and enhance students learning.

3.2 OBJECTIVES

After you study this lesson, you will be able to:

1. Define pedagogic content knowledge.
2. Describe pedagogic content knowledge analysis in mathematics.
3. Describe framework of pedagogic content knowledge analysis in mathematics.
4. Define integers, rational number, real number, polynomials, quadratic equations, and congruency of triangles
5. Tell properties of integers, rational number, real number, polynomials, quadratic equations, and congruency of triangles.
6. Solve problems related tointegers, rational number, real number, polynomials, quadratic equations, and congruency of triangles.

3.3 PEDAGOGIC CONTENT KNOWLEDGE ANALYSIS FOR – FACTS, CONCEPTS, GENERALIZATIONS, AND PROCEDURES

Pedagogic content knowledge analysis is essential for effective teaching and student learning. When teachers' subject matter and pedagogical content knowledge are in concert, instruction is likely to be most effective and reflect both the art and the science of teaching (Alexander & Jetton, 2000; Carney &Indrisano, 2013).

According to Shulman (1986), pedagogical content knowledgeis a second kind of content knowledge, which goes beyond knowledge of subject matter per se to the dimension of subject matter knowledge for teaching. In pedagogical content knowledge, Shulman (1986) included -

i. The most regularly taught topics in one's subject area, the most useful forms of representation of those ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations—in a word, the ways of representing and formulating the subject that make it comprehensible to others.

ii. An understanding of what makes the learning of specific topics easy or difficult: the conceptions and preconceptions that students of different ages and backgrounds bring with them to the learning of those most frequently taught topics and lessons. If those preconceptions are misconceptions, which they so often are, teachers need knowledge of the strategies most likely to be fruitful in reorganizing the understanding of learners, because those learners are unlikely to appear before them as blank slates. (p. 6-7).

According to Shulman (1987), reflection and evaluation are important elements in teachers' development of pedagogic content knowledge.

According to Koehler, Mishra, and Cain (2013),pedagogic content knowledge covers the core business of teaching, learning, curriculum, assessment, and reporting, such as the conditions that promote learning and links among curriculum, assessment, and pedagogy.

According to Gess-Newsome (2015), pedagogic content knowledge is the knowledge that teachers bring forward to design and reflect on instruction.

According to Nilsson and Karlsson (2019), pedagogic content knowledge is the knowledge that teachers activate when they plan particular lessons on a specific topic or when they reflect upon them afterwards.

A Framework for Analyzing Teachers' Pedagogical Content Beliefs given by Peterson, Fennema, Carpenter, and Loef (1989) can be help mathematics teachers indeveloping pedagogic content knowledge analysis for facts, concepts, generalizations, and procedures. The framework is:

1. **Children construct their own mathematical knowledge.** Cognitive science researchers have shown that children develop informal systems of mathematics outside the classroom. Children do not simply absorb what they are taught; they structure and interpret the presented mathematics curriculum and instruction in light of their existing knowledge. The assumption might be represented as a continuum, ranging from *children actively construct their own knowledge at one end to children passively receive mathematical knowledge from the teacher or others at the other end*.

2. **Mathematics instruction should be organized to facilitate children's construction of knowledge.** A second assumption related to the first that mathematics instruction should be organized to facilitate children's knowledge construction. A contrasting assumption might be that mathematics instruction should be organized to facilitate the teacher's clear presentation of knowledge.

3. **Children's development of mathematical ideas should provide the basis for sequencing topics for instruction.** A third assumption related to the first two is that children's naturalistic development of mathematical ideas should provide the basis for sequencing topics for instruction. A contrasting notion might be that the structure of mathematics should provide the basis for sequencing topics for instruction. If the structure of mathematics were to provide the basis for sequencing topics for instruction, it follows that instruction would begin with teaching formal mathematical symbolism and procedures. However, basing sequence on the naturalistic development of ideas acknowledges that children use direct modeling strategies and counting strategies before they can come to understand and use the more abstract number facts and written symbolism that accompany number facts.

4. **Mathematical skills should be taught in relation to understanding and problem solving.** A fourth assumption is that mathematical skills should be taught in relation to understanding and problem solving. A contrasting perspective would be that mathematical skills should be taught as discrete components in isolation from understanding and problem solving. Both perspectives assume that the learning of mathematics skills, understanding, and problem solving are all important goals of mathematics instruction. However, the two propositions make very different statements both about their relationships and about the most effective ways to achieve these goals.

Also, Nilsson and Karlsson (2019) pointed out that in order to analysethe teaching performance, focusing attention to 'critical incidents' is necessary. The critical incidents are the incidents where teachers thought they had either failed or succeeded in their teaching. Study by Nilsson and Karlsson (2019) has important implications for pedagogic content knowledge analysis for facts, concepts, generalizations, and procedures. Nilsson and Karlssonpointed out that development of pedagogic content knowledgerequires the teacher to reflect upon how to teach a specific topic in order to promote students' learning. It prompts the teacher to articulate what is called 'Big Ideas' relating to queries that include:

i. What students should learn about each big idea;

ii. Why it is important for students to know these ideas;

iii. Students' possible difficulties with learning the ideas; and

iv. How these ideas fit in with the knowledge the teacher holds about that content.

With the help of these queries, *what students should learn about each big idea: why it is important for students to know these ideas; students' possible difficulties with learning the ideas; and how these ideas fit in with the knowledge the teacher holds about that content,* and framework, *Children construct their own mathematical knowledge; Mathematics instruction should be organized to facilitate children's construction of knowledge; Children's development of mathematical ideas should provide the basis for sequencing topics for instruction; and Mathematical skills should be taught in relation to understanding and problem solving*, mathematics teachers can develop pedagogic content knowledge analysis for various facts, concepts, generalizations, and procedures of mathematics.

Check Your Progress

Notes : (a) Write your answers in the space given below.

(b) Compare your answers with those given at the end of the unit.

1. Which reflective questions can help a mathematics teacher in developing pedagogical content knowledge analysis for facts, concepts, generalizations, and procedures of mathematics?

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3.4 KNOWLEDGE AND TEACHING OF INTEGERS, RATIONAL NUMBER, REAL NUMBER, POLYNOMIALS, QUADRATIC EQUATIONS, AND CONGRUENCY OF TRIANGLES.

INTEGERS

Integers are signed numbers made of: a) a sign, + for positive numbers and – for negative numbersb) an absolute value, that is a natural number. In other words, integers form a collection of numbers which contains whole numbers and negative numbers.

i. For example: -9.The sign is ? . The absolute value is 9.

ii. 5. The sign is + (by default). The absolute value is 5.

Properties of Integers:

i. 0 is considered neither positive nor negative. However, some generalizations require us to consider the following numbers: +0 and -0. Still, these two instances of the number zero are equal to 0.

ii. The set of all integers can be represented by using the set notation as:

$Z = \{...,-3,-2,-1,0,+1,+2,+3,...\}$.

iii. The set of all positive integers can be represented by using the set notation as:

Z_+ or $Z^+ = \{+1,+2,+3,...\}$.

iv. The set of all negative integers can be represented by using the set notation as: Z_- or $Z^- = Z = \{...,-3,-2,-1\}$.

v. There is no biggest or smallest integer but there are infinitely many integers.

$+\infty$ is a symbol for infinite positive integers

$-\infty$ is a symbol for infinite negative integer

vi. Integers are closed under addition that is for any two integers 'a' and 'b', a + b is an integer.

vii. Integers are closed under subtraction that is for any two integers ‘a’ and ‘b’, a – b is an integer.

viii. Addition is commutative for integers that is for any two integers ‘a’ and ‘b’, a + b = b + a.

ix. Subtraction is not commutative for integers that is for any two integers ‘a’ and ‘b’, a – b = b – a.

x. Addition is associative for integers that is for any integers ‘a’, ‘b’, and ‘c’, a + (b + c) = (a + b) + c.

xi. 0 is additive identity for integers as for any integer ‘a’, a + 0 = a = 0 + a.

xii. For a positive integer ‘a’ and a negative integer ‘–b’, a × (– b) = – (a × b).

xiii. For any two negative integers ‘–a’ and ‘–b’, (– a) × (– b) = a × b.

xiv. Integers are closed under multiplication that is a × b is an integer, for all integers ‘a’ and ‘b’.

xv. Multiplication is commutative for integers that is a × b = b × a where ‘a’ and ‘b’ are any two integers.

xvi. For any integer ‘a’, a × 0 = 0 × a = 0.

xvii. 1 is the multiplicative identity for integers.

xviii. One can getadditive inverse of an integer ‘a’by multiplying ‘–1’ to ‘a’ that is a × (–1) = (–1) × a = – a.

xix. The associative property of multiplication does not depend on the grouping of the integers that is any three integers ‘a’, ‘b’, and ‘c’, (a × b) × c = a × (b × c).

xx. a × (b + c) = a × b + a × c where ‘a’, ‘b’, and ‘c’ are any integers.

xxi. a ÷ (– b) = (– a) ÷ b where ‘a’, ‘b’ are integers and b ≠ 0.

xxii. a ÷ 1 = a, and a ÷ (–1) = – a where ‘a’ is an integer

RATIONAL NUMBERS

A rational number (or a rational fraction) is a number which can be put in the form p/q, where p and q are integers, and q is not zero. Rational numbers are denoted by Q.

$$Q = \left\{\frac{p}{q} : p, q \in Z \text{ and } q \neq 0\right\}$$

The rational numbers can be represented in decimal form also. The decimal representations of some rational numbers are terminating, or recurring.

A rational fraction p/q in lowest terms has a terminating decimal expansion if and only if the integer q has no prime factors other than 2 and 5.

Properties of Rational Numbers

i. All integers and fractions are rational numbers.

ii. Let Q be a rational number. If Q is multiplied or divided by an integer other than 0 then we get a rational number which is said to be equivalent to Q.

iii. Rational numbers are can be as positive, zero or negative rationalnumbers. For example, $\frac{4}{5}$, 0, and $\frac{-4}{5}$ are positive, zero and negative rational numbers, respectively.

iv. There are infinite of rational numbers between two rationalnumbers.

v. The product of two or more rational numbers is a rational number.

vi. The division of a rational number by a non-zero rational number is a rational number.

vii. Rational numbers are closed under addition, subtraction, and multiplication.

viii. The operations addition and multiplication are (a) commutative and (b) associative, for rational numbers.

ix. The rational number 0 is the additive identity for rational numbers.

x. The rational number 1 is the multiplicative identity for rational numbers.

xi. The additive inverse of the rational number $\frac{p}{q}$ is $-\frac{p}{q}$ and vice-versa.

xii. Distributive property of rational numbers states that for all rational numbers p, q and r, p(q + r) = pq + pr and p(q – r) = pq – pr.

REAL NUMBERS

The real numbers are all of the points on the number line. The set of real numbers consists of both the rational numbers and the irrational numbers. Rational numbers are terminating or recurring decimal while Irrational numbers are neither terminating nor recurring.

Real numbers are 'numbers' with decimal expansions which may or may not be recurring. The set of real numbers is denoted by R.

$R = Q \cup \overline{Q}$

For example:

a. There is a real number √2.

√2 = 1.4142135623730950488016887242096980785696 7...........

b. There is a real number ? whose value is known to nearly 3 trillion places.

π = 3.14159265358979323846264338327950288419716939937 5........

Properties of Real Numbers

i. Real numbers are commutative under addition that is for all real numbers x and y, x + y = x.

ii. Real numbers are commutative under associativity that is for all real numbers x, y, andz, x +(y +z) = (x +y) +z.

iii. The number 0 (zero) is additive identity of real numbers such that, for all x, x + 0 =x.

iv. The additive inverse of real numbers is number with minus sign that is for each x, there isa number -x such that x + (-x) = 0.

v. Real numbers are commutative under multiplication that is for all real numbers x and y, $x \times y = y \times x$.

vi. Real numbers are associative under multiplication that is for all real numbers x, y, and z, $x \times (y \times z) = (x \times y) \times z$.

vii. There is a number 1, called the multiplicative identity, which is different from 0, such that, for all x, $x \times 1 = x$.

viii. For each $x \neq 0$, there is a number $\frac{1}{x}$ such that $(\frac{1}{x} \times x) = 1$. Here $\frac{1}{x}$ is multiplicative inverse.

ix. Multiplication of real numbers is distributive, for all x, y, and z, $x \times (y + z) = x \times y + x \times z$.

x. A set S of real numbers is convex if, whenever x_1 and x_2 belong to S and y is a number such that $x_1 < y < x_2$, then y belongs to S as well.

Check Your Progress

Notes : (a) Write your answers in the space given below.

(b) Compare your answers with those given at the end of the unit.

2. Prove that $\log_2 5$ is not rational.

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3. Find three rational numbers between $\frac{1}{3}$ and $\frac{1}{2}$.

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4. Plot √2 on the number line.

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5. Write three properties of real numbers.

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POLYNOMIALS

Polynomial is made up of two words poly- (meaning "many") and -nomial (in this case meaning "term"). Thus polynomial means "many terms".A polynomial is as expression such as $5x^4 - 2x^3 + 8x + 9$ or $\frac{1}{4}x^2 - \sqrt{2}x$? 3. There may be anynumber of terms, but each term must be a multiple of a whole number power of x. A polynomial may have following terms:

Variable - a variable is denoted by a symbol (x, y, z, etc.) that can take any real value. For example, $5x^4 - 2x^3 + 8x + 9$ is a polynomial in x.

Constant –a constant is one that can take only one specific value. For example, 2, 3, 4, 100, 34 etc. are constants. In $5x^4 - 2x^3 + 8x + 9$, 9 is constant.

Degree/Exponent –Thedegreeof a polynomial in x is equal to the highest power of x in the polynomial. For example, in $5x^4 - 2x^3 + 8x + 9$, the highest degree of x is 4. Thus the degree of polynomial $5x^4 - 2x^3 + 8x + 9,$ is 4.

Coefficient - Each term of a polynomial has a coefficient. For example,in $5x^4 - 2x^3 + 8x + 9$, the coefficient of $5x^4$ is 5, the coefficient of $2x^3$ is 2, the coefficient of $8x\ is\ 8$, and coefficient $x^0$of 9 (as x^0 =1).

The general polynomial is of the form p(x) = $a_n x^n - a_{n-1} x^{n-1} + ... + a^1 x + a_0$, where $a_n \neq 0$ and n is a whole number. The coefficients are, in general, real numbers.

Polynomial can be classified according to their degree.

Zero Polynomial	*Constant Polynomial*	*Linear Polynomial*	*Quadratic Polynomial*	*Cubic Polynomial*	*Quartic Polynomial*	*Quintic Polynomial*
The constant polynomial 0. $p(x) = 0$	A polynomi al having only a non-zero constant is called a constant polynomial. Here, 2, 7, 8 are constant polynomials.	A polynomial of degree 1. Here, $4 + 9$ is a linear polynomial.	A polynomial of degree 2. Here, $4^2 + 9$ is a quadratic polynomial.	A polynomial of degree 3. Here, $4^3 + 9$ is a cubic polynomial.	A polynomial of degree 4. Here, $4^4 + 9$ is a quartic polynomial.	A polynomial of degree 5. Here, $4^5 + 9$ is a quintic polynomial.

Remainder Theorem

The division of polynomials can be made easy with the help of remainder theorem. Here we will study remainder theorem.

Remainder Theorem : Let p(x) be any polynomial of degree greater than orequal to one and let c be any real number. If p(x) is divided by the linearpolynomial x – c, then the remainder is p(c).

Proof : Let p(x) be any polynomial with degree greater than or equal to 1.

Suppose that when p(x) is divided by x – c, the quotient is q(x) and the remainder is r(x), i.e.,

$$p(x) = (x - c)\ q(x) + r(x)$$

Since the degree of x–c is 1 and the degree of r(x) is less than the degree of x–a, the degree of r(x) = 0. This means that r(x) is a constant, say r.

So, for every value of x, r(x) = r.

Therefore, $p(x) = (x - c)\ q(x) + r$

In particular, if x = c, this equation gives us

$p(c) = (c - c)\ q(c) + r$

= r, Hence Proved

Applications of remainder theorem

Suppose f(x)is a polynomial of degree ≥ 1 then

(a) The remainder when f(x) is divided by (x+a) is f(-a)

(b) The remainder when f(x) is divided by (ax +b) is $f\left(\frac{-b}{a}\right)$

(c) The remainder when f(x) is divided by (ax – b) is $f\left(\frac{b}{a}\right)$

The Factor Theorem

The Factor Theorem is a result of the Remainder Theorem. It helps us analyze polynomial equations. It tells us how the zeros of a polynomial are related to the factors.

Factor Theorem: If p(x) is a polynomial of degree n> 1 and c is any real number, then

(i) x – c is a factor of p(x), if p(c) = 0, and

(ii) Conversely, if x – c is a factor of p(x) then p(c) = 0.

Proof : We know that p(x)=(x – c) q(x) + p(c) (By Remainder Theorem)

(i) If p(c) = 0, then p(x) = (x – c) q(x), which shows that x – c is a factor of p(x).

(ii) Since x – c is a factor of p(x), p(x) = (x – c) g(x) for some polynomial g(x). In this case, p(c) = (c – c) g(c) = 0.

Application of factor theorem

(a) (x – a) is a factor of $x^n - a^n$ for any $n \in N$ (any natural number)

(b) (x + a) is a factor of $x^n - a^n$ if n is even number

(c) (x + a) is a factor of $x^n + a^n$ if n is odd number

QUADRATIC EQUATIONS

A quadratic equation is a polynomial with degree '2'. The standard form of a quadratic equation is $ax^2 + bx + c = 0$, where a, b and c are numbers and $a \neq 0$.

The examples of quadratic equations are :

$$x^2 + 3x - 4 = 0,4x - 45x^2 + 400 = 0,43x^2 - 4 = 0 \text{ etc.}$$

Quadratic equations can be used with four methods:

A. Solution by factorisation

B. Solution by completing the square

C. Solution using a quadratic formula

D. Solution using graphs

A. Solution by factorization

Step I : Write the equation in standard form.

Step II : Factorize the quadratic equation on the left hand side if possible.

Step III : The left hand side will be the product of two linear factors. Then equate each of the linear factor to zero and solve for values of x. These values of x give the solution of the equation.

Example : Solve the equation $2x^2 - 7x = -6$

Solution : The given equation is $2x^2 - 7x = -6$

Standard form of the equation is $2x^2 - 7x + 6 = 0$

Factorizing L.H.S., we get

$2x - 4x - 3x + 6 = 0$ [Because, – 4 – 3 = –7, and (– 4) × (– 3) = 12]

$2x\ (x — 2)\ x - 3(x - 2) = 0$

$(2x - 3)(x - 2) = 0$

Equating each of the linear factors to zero, we get

$2x - 3 = 0$ or $x - 2 = 0$

$2x = 3$ or $x = 2$

$x = \frac{3}{2}$

B. Solution by completing square

Finding solution of a quadratic equation by completing square was the most commonly used method of solution. While using this method, teacher assumes that students have knowledge of two identities, namely, referred to as perfect squares:

$$a^2 + 2ab + b^2 = (a + b)^2 \text{ and } a^2 - 2ab + b^2 = (a - b)^2.$$

The procedure of finding solution of a quadratic equation by completing square is:

Step I : Write the quadratic equation is standard form.

Step II : Divide both sides of the equation by the co-efficient of x^2 if it isnot already 1.

Step III : Shift the constant term to the R.H.S.

Step IV : Add the square of one-half of the co-efficient of x to both sides.

Step V : Write the L.H.S as complete square and simplify the R.H.S.

Step VI : Take the square root on both sides and solve for x.

Example : Solve $3x^2 + 1 = 5x$

Solution:

$3x^2 - 5x + 1 = 0$ (By writing in the standard form)

$x^2 - \frac{5}{3}x = -\frac{1}{3}$ (By dividing the equation by 3 and shifting the constant term to the other side)

$x^2 - \frac{5}{3}x + \frac{25}{36} = -\frac{1}{3} + \frac{25}{36}$ (By adding $[\frac{1}{2}(\frac{5}{3})]^2$ to both sides)

$(x - \frac{5}{6})^2 = \frac{25}{36} - \frac{1}{3}$

$(x - \frac{5}{6})^2 = \frac{13}{36}$ (By completing square and simplifying R.H.S.)

Or $x - \frac{5}{6} = \pm(\frac{\sqrt{13}}{6})$ (By taking square root on both sides)

$x - \frac{5}{6} = \frac{\sqrt{13}}{6}$ or $x = \frac{5}{6} - \frac{\sqrt{13}}{6}$

$x = \frac{\sqrt{13}}{6} + \frac{5}{6}$ $x = \frac{5-\sqrt{13}}{6}$

$x = \frac{\sqrt{13}+5}{6}$

$x = \frac{5+\sqrt{13}}{6}$

C. Solution using a quadratic formula

By applying method of completing the square to the general quadratic equation, we obtain the well-known quadratic formula.

The general quadratic equations

$$ax^2 + bx + c = 0 \qquad a \neq 0$$

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$ (By dividing throughout by a)

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$ (By shifting constant to R.H.S.)

$$x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} = \frac{b^2}{4a^2} - \frac{c}{a}$$ $\left[\text{By adding } \left(\frac{b}{4a}\right)^2 \text{ to both sides}\right]$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$ [by completing square and simplifying R.H.S.]

$$x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{2a}$$ [By taking square root of both sides]

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = -\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Which is called Quadratic formula.

E. Solution of quadratic equation by graph

A quadratic equation in the form $y = ax^2 + bx + c$ has a distinctive parabola shape. It will have a minimum vertex with the curve opening upward when the value of a is greater than zero ($a>0$). It will have a maximum vertex with the curve opening downward when the value of a is less than zero ($a<0$).

Note that the value of a cannot be equal to zero, as this would mean that there is no x^2-term, and as such it would not be a quadratic equation.

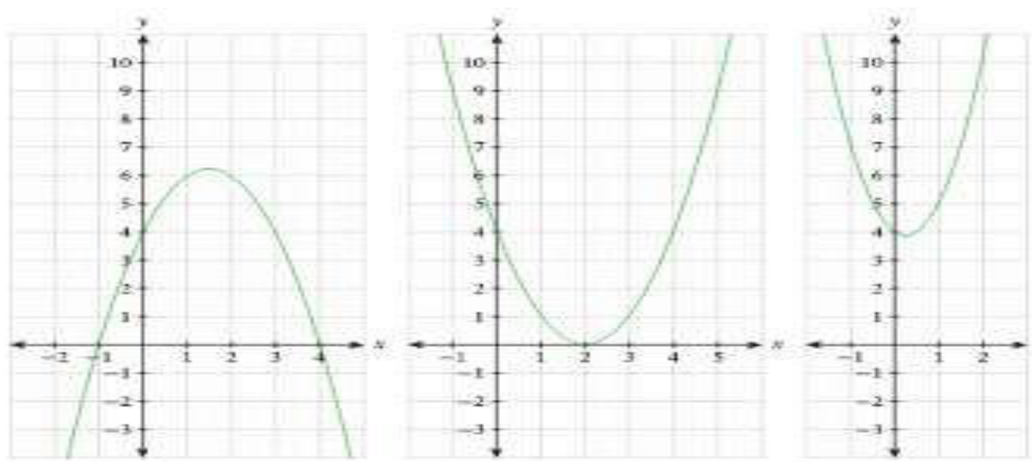
The graph of a quadratic equation is symmetrical about the vertex.

The y-intercept, the coordinate where the parabola crosses the y-axis, of the function.

$y = ax^2+bx+c$ will always be at (0, c).

The x-intercepts, where the curve crosses the x-axis, will be the points at which y=0. These are also called the roots of the equation. In a graph, we can indentify these points by inspecting the graph.

It is useful to remember that a quadratic equation will have up to two roots. An equation with two roots will have a graph which crosses the x-axis twice, a repeated root will mean that the graph has a vertex on the x-axis, and an equation with no roots will have a graph that is above the x-axis.



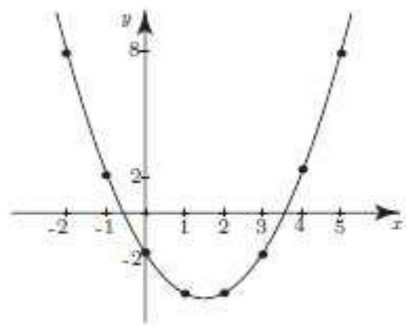
In the graphs above, the first graph has two roots, the middle graph has one root, where the graph touches the x-axis, and the last graph has no roots.

Example:

Suppose we wish to solve $x^2 - 3x - 2 = 0$.
We consider $y = x^2 - 3x - 2$ and produce a table of values so that we can plot a graph.

x	-2	-1	0	1	2	3	4	5
x^2	4	1	0	1	4	9	16	25
$-3x$	6	3	0	-3	-6	-9	-12	-15
-2	-2	-2	-2	-2	-2	-2	-2	-2
$x^2 - 3x - 2$	8	2	-2	-4	-4	-2	2	8

From this table of values a graph can be plotted, or sketched as shown in Figure 4. From the graph we observe that solutions of the equation $x^2 - 3x - 2 = 0$ lie between -1 and 0, and between 3 and 4.

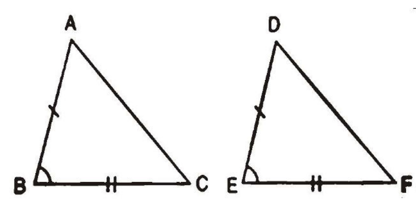


CONGRUENCY OF TRIANGLES

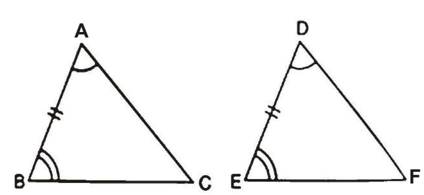
Two triangles are called congruent triangles if they are equal in all respects. Congruent triangles are equal with respect to shapes and sizes. The corresponding parts in congruent triangles are equal and we write in short 'CPCT' for corresponding parts of congruent triangles.

Criteria for Congruence of Triangles:

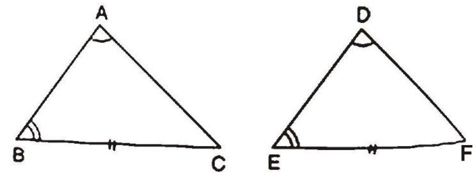
Side-Angle-Side (SAS)–Two triangles are congruent, if two sides and the included angle of one triangle are equal to the two sides and the included angle of the other triangle.



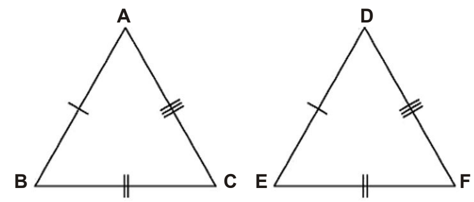
Angle-Side-Angle (ASA)–Two triangles are congruent, if two angles and the included side of one triangle are equal to two angles and the included side of other triangle.



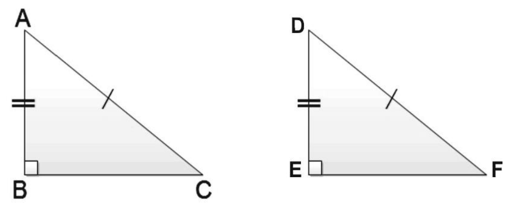
Angle-Angle-Side (AAS) - Two triangles are congruent, if any two pairs of angles and one pair of corresponding sides are equal.



Side-Side-Side (SSS) -Two triangles are congruent if threesides of one triangle are equal to three sides of the other triangle.



Right Angle-Hypotenuse-Side (RHS) -Two right angled triangles are congruent, if the hypotenuse and one side of one triangle are equal to the hypotenuse and a side of the other triangle.



Check Your Progress

Notes : (a) Write your answers in the space given below.

(b) Compare your answers with those given at the end of the unit.

6. Find the remainder when $p(x) = x^4 - 3x^2 - 10x + 2$ is divided by $(2x - 1)$.

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7. State Factor theorem.

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3.5 LET US SUM UP

In the present lesson, we studied that pedagogical content knowledge analysis is a powerful item for bringing about effectiveness in teaching and students learning. Pedagogical content knowledge analysis asks a teacher to find the answers of queries, like, What students should learn about each topic; Why it is important for students to know this topic; Students' possible difficulties with learning the topic; and How this topic fit in with the knowledge the teacher holds about that content. Also we learnt about knowledge and teaching of concepts, namely, integers, rational number, real number, polynomials, quadratic equations, and congruency of triangles.

3.6 LESSON–END EXERCISE

1. What is the framework and queries for pedagogic content knowledge analysis for facts, concepts, generalizations, and procedures of mathematics?
2. Define integers, rational number, real number, polynomials, quadratic equations, and congruency of triangles
3. Tell properties of integers, rational number, real number, polynomials, quadratic equations, and congruency of triangles.
4. Solve problems related to integers, rational number, real number, polynomials, quadratic equations, and congruency of triangles.
5. Describe methods of solving quadratic equations.
6. Write five rational numbers greater than –3.
7. Solve $x^4 + x^3 - 2x^2 - 7x + 1 = 0$
8. Prove that $\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$ if $b \neq 0$ and $d \neq 0$.
9. For what value of K the equation $(4+k)\,x^2 + 2(k-2)\,x + 8k + 1 = 0$ will be a perfect square.
10. Prove that the integers of the form $3n + 1$ are not closed under addition.

11. Prove that the rational number 1/3 can be written in rational form p/q in infinitely many ways.

12. If the diagonals of a quadrilateral bisect each other at right angle then prove that the quadrilateral is a rhombus.

13. PQ an RS are equal perpendiculars to a line segment PR. If PQ and RS are on different sides of PR, prove that SQ bisects PR.

3.7 SUGGESTED FURTHER READINGS

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3.8 ANSWERS TO CHECK YOUR PROGRESS

1. a. What students should learn about each big idea/topic;

   b. Why it is important for students to know these ideas/topic/sub-topics;

   c. Students' possible difficulties with learning the ideas; and

   d. How these ideas fit in with the knowledge the teacher holds about that content.

2. As with the proof that 2 is irrational, we begin by supposing the contrary.

   Suppose that $\log_2 5 = \frac{p}{q}$, where p and q are whole numbers. We can rewrite this statement without logarithms as $5 = 2^{p/q}$. Raising both sides to the power gives, $5^q = 2^p$.

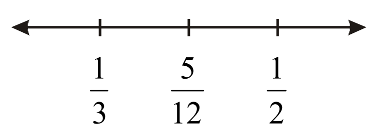
   Now this 'equation' is impossible, since the left hand side is odd, while the right hand side is even. Thus, $\log_2 5$ is irrational.

3. We find the mean of the given rational numbers :

$$\left(\frac{1}{3}+\frac{1}{2}\right)\div 2=\left(\frac{2+5}{6}\right)\div 2=\frac{5}{6}\times\frac{1}{2}=\frac{5}{12}$$

So $\frac{5}{12}$ lies between $\frac{1}{3}$ and $\frac{1}{2}$,

$$\frac{1}{3}<\frac{5}{12}<\frac{1}{2}$$

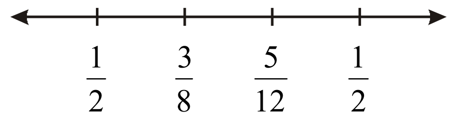


We from this find another rational number between $\frac{1}{3}$ and $\frac{5}{12}$.

For this we again find the mean of $\frac{1}{3}$ and $\frac{5}{12}$.

$$\left(\frac{1}{3}+\frac{5}{12}\right)\div 2=\left(\frac{4+5}{12}\right)\div 2=\frac{9}{12}\times\frac{1}{2}=\frac{9}{24}=\frac{9}{24}=\frac{3}{8}$$

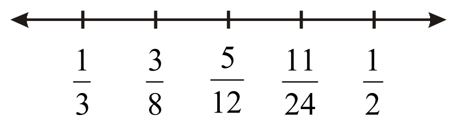
So $\frac{1}{3}<\frac{3}{8}<\frac{5}{12}<\frac{1}{2}$



Now we will find a rational number between $\frac{5}{12}$ and $\frac{1}{2}$.

$$\left(\frac{5}{12}+\frac{1}{2}\right)\div 2=\left(\frac{5+6}{12}\right)\div 2=\frac{11}{12}\times\frac{1}{2}=\frac{11}{24}$$

Thus we get, $\frac{1}{3}<\frac{3}{8}<\frac{5}{12}<\frac{11}{24}<\frac{1}{2}$

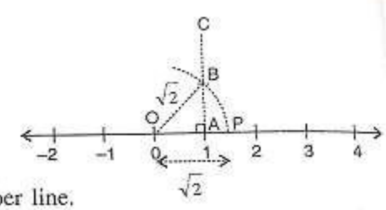


Thus, $\frac{3}{8}$, $\frac{5}{12}$ and $\frac{11}{24}$ are three rational numbers between $\frac{1}{3}$ and $\frac{1}{2}$

4. **(b) For irrational numbers :**

(1) To represent $\sqrt{2}$ on the number line :

Steps :



1. Mark 0 of the number line at point O.
2. On the same number line, mark 1 as point A.
   i.e. take OA = 1 unit
3. At point A, draw AC perpendicular to the number line.
4. From AC, cut AB = 1 unit = OA, then join O and B.
   Using Pythagoras theorem, we get :

$$OB^2 = OA^2 + AB^2$$
$$= (1)^2 + (1)^2 = 1 + 1 = 2 \quad \therefore OB = \sqrt{2}$$

5. i. Real numbers are commutative under addition that is for all real numbers x and y, x + y = x.

   ii. Real numbers are commutative under associativity that is for all real numbers x, y, and z, x +(y +z) = (x +y) +z.

   iii. The number 0 (zero) is additive identity of real numbers such that, for all x, x + 0 = x.

6. Write $p(x) = (2x - 1)q(x) + r$, so we can obtain r by substituting $x = ½$ in both sides.

   Hence the remainder is $r(½) = \frac{1}{16} - \frac{3}{4} - 5 + 2 = - -\frac{59}{16}$.

7. If p(x) is a polynomial of degree n> 1 and c is any real number, then

   (i) x – c is a factor of p(x), if p(c) = 0, and

   (ii) Conversely, if x – c is a factor of p(x) then p(c) = 0.

—×—

LESSON NO. 4 UNIT-II
OBJECTIVES AND METHODS OF MATHEMATICS TEACHING

STRUCTURE

4.1 INTRODUCTION

What are the main goals of mathematics education in schools? Simply stated, there is one main goal—the mathematisation of the child's thought processes. In the words of David Wheeler, it is "more useful to know how to mathematise than to know a lot of mathematics (NCERT, 2006)."

The present chapter may help in getting awareness about objectives of teaching mathematics and Anderson's revised Bloom's taxonomy of instructional objectives.

4.2 OBJECTIVES

After you study this lesson, you will be able to:

1. List the objectives of teaching mathematics.
2. Write disciplinary, utilitarian, cultural, social, and recreational objectives of mathematics.
3. Explain the Anderson's revised Bloom's taxonomy of instructional objectives.
4. Tell objectives of teaching arithmetic, algebra, and geometry.

4.3 OBJECTIVES OF TEACHING MATHEMATICS

The Programme for International Student Assessment (PISA) has stated that the objective of mathematics teaching of 15 year olds i.e. secondary stage students is to have the knowledge and skills in mathematical literacy. PISA has defined mathematical literacy as *an individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgments and to use and engage with mathematics in ways that meet the needs of that individual's life as a constructive, concerned and reflective citizen.* The National Curriculum Framework (NCF) (2005) suggested that developing children's abilities for mathematisation is the main goal of mathematics education. According to NCF (2005), the objectives for School Mathematics are:

i. Children learn to enjoy mathematics rather than fear it.

ii. Children learn important mathematics: Mathematics is more than formulas and mechanical procedures.

iii. Children see mathematics as something to talk about, to communicate through, to discuss among themselves, to work together on.

iv. Children pose and solve meaningful problems.

v. Children use abstractions to perceive relation-ships, to see structures, to reason out things, to argue the truth or falsity of statements.

vi. Children understand the basic structure of Mathematics: Arithmetic, algebra, geometry and trigonometry, the basic content areas of school Mathematics, all offer a methodology for abstraction, structuration and generalisation.

vii. Teachers engage every child in class with the conviction that everyone can learn mathematics.

Moreover, the objectives of teaching mathematics can be classified as under:

A. Disciplinary objectives.

B. Utilitarian objectives.

C. Cultural objectives.

D. Social objectives.

E. Recreational objectives.

A. Disciplinary Objectives

To develop students' cognitive powers and disciplinary minds

An individual must be able to adjust effectively in his environment and make best use his capacities. And, this is possible if the subjects taught at school level have the power to discipline students' mind. The mathematics is one such subject that aims to contribute significantly towards the development of a disciplined mind.

Mathematics is a proper workout for the human mind. The way physical exercise trains our muscles, mathematics exercises can train our brain. As in the words of Schultze, "Mathematics is primarily taught on account of mental training it afford, and only secondarily on account of knowledge of facts it imparts." Also, according to Locke, "Mathematics is a way to settle in the mind a habit of reasoning." An individual with a capacity to reason will act intelligently in different life situations.

Furthermore, the teaching mathematics fulfills its disciplinary objective by training students' power of concentration; ability to think creatively; problem solving; ability to be precise in thinking and action; and will power.

B. Utilitarian Objectives

To enable the students to make use of mathematics effectively in their day-to-day activities.

The progress and development of the modern world cannot be conceived without mathematics. The technological advances made by human beings are being super-structured upon mathematics. The utilitarian objective of teaching mathematics is so evident that we cannot assume any activity of life where there is no mathematics. In our day-to-day life mathematics helps us in construction work, getting jobs and becoming proficient in one's vocation, preparation of budgets, calculation of wages, making schedule of trains and airplanes, etc. Also, the utilitarian objective of teaching mathematics helps various sciences to become more exact. As Kant argued that "A Science is exact only in so far as it employs Mathematics." moreover, teaching of mathematics not only helps various sciences to become more exact, mathematics is in fact is necessary for understanding different subjects. As Roger Bacon pointed out, "Mathematics is the gate and key of the sciences. ... Neglect of mathematics works injury to all knowledge, since he who is ignorant of it cannot know the other sciences or the things of this world." In short, we can say that we cannot assume any life activity where the utilitarian objective of teaching mathematics does not fulfills.

C. Cultural Objectives

To help students understand the contribution of mathematics in the development of culture and civilization.

Culture is what a group of people do in unison. This is about religious orientations, social customs, table manners, beliefs about right and wrong, behavior interpretations that distinguishes a group of peoples from others. And, mathematics plays a major role in the progression and transmission of the culture of any community or a group. Mathematics not only acquaints us about our culture but also makes significant contributions in development of our culture. According to D'Entremont, "Mathematics is an integral component of all cultural contexts and the significance of all cultural contexts is influenced by the interpretation the individual makes within that culture." Mathematics changes our view of life by facilitating our reasoning and thinking abilities. The history of mathematics has suggested that cultural norms of ancient and medieval periods have been influenced by the mathematics of those times. The superiority or inferiority of mathematics of a particular society is reflected in those folkways of the society. Hogben pointed out that "Mathematics is the mirror of civilization." However, the relationship between culture and mathematics may be a reciprocal one, as the ethnomathematicians believe that mathematics is cultural creation. Whatever may be the case, mathematics is essential aspect of cultural evolution and transmission.

D. Social Objectives

To imbibe in students the necessary social values.

Man is a social animal and the innumerable needs of that social animal are taken care by mathematics each second of a day. The present fabric of society has been weaved around mathematics. The societal needs of inventing high technology products have been conceived through mathematics only. The change in our lifestyle brought about by advances in the field of science and technology is due to mathematics only. The computer programming which is backbone of our society is written in mathematical language. The doctors, engineers, physicists, librarians, geologists, sociologists, economists, astronomers, religious head, all need mathematical literacy to survive and progress. As Napolean said that, "The progress and improvement of mathematics are linked to the prosperity of the state." The society has so much dependence upon mathematics that any hitch in teaching mathematics properly can bring our society on a standstill. The social utility of mathematics can be seen in the role played by mathematics in our trades and commerce, material development, organizational structure of society, advancements in outer space, and boundless comforts of life. Teaching of mathematics can help our society to sustain and progress. Thus, we need to teach mathematics for its social objective also.

E. Recreational Objectives

To help students enjoy mathematics while understanding and doing mathematics.

'All work and no play makes Jack a dull boy' proverb reminds us that mathematics need to be taught for its recreational value also. The teaching of mathematics should be so oriented that mathematics accomplishes its recreational objectives effectively. Mathematical puzzles and games can be used effectively to make mathematics a fun oriented activity. Magic squares and geoboard can also be used for making mathematics an enjoying activity. This may also help in understanding mathematics and creating interest in learning higher mathematics. Such activities can be organized in mathematics club that help students to know about the recreational attribute of mathematics. The mathematics has recreational objective from the ancient times. The Egyptian's in 800 BC, had a problem,"Seven houses; in each are 7 cats; each cat kills 7 mice; each mouse would have eaten 7 ears of spelt; each ear of spelt will produce 7 hekat. What is the total of all of them?" Also, some activities can be like,

A. Suppose you have a barrel of water, a seven cup can, and an eight cup can. The cans have no marking on them to indicate a smaller number of cups such as 3 cups. How can you measure nine cups of water using only the seven-cup can and the eight-cup can. (Balka, 1974)

B. Magic Square

9	3	22	16	15
2	21	20	14	8
25	19	13	7	1
18	12	6	5	24
11	10	4	23	17

C. Pie Cut

With one straight cut you can slice a pie into two pieces. A second cut that crosses the first one will produce four pieces, and a third cut can produce as many as seven pieces. What is the largest number of pieces that you can get with six straight cuts?

Thus, the teaching of mathematics should strive to accomplish Disciplinary, Utilitarian, Cultural, Social, and Recreational objectives. In order to bring about the clarification for teachers, these objectives can be further simplified as knowledge objectives, skill objectives, application objectives, appreciation objectives, and attitude objectives.

Knowledge Objectives

A teacher of mathematics should help students acquire the following knowledge and understanding objectives

a. to understand mathematical axioms, postulates, formulae, principals, concepts, facts and processes.

b. to get knowledge of various graphs.

c. To develop the concept of directed number, set function, mapping inequalities and graph.

d. To deal with arithmetic, algebraic, and geometric entities.

e. To understand mathematical language and symbols.

f. To know about the contribution of mathematicians in the development of civilization and space exploration.

g. To understand inter-relationship and inter-dependence between various topics and branches of mathematics.

h. To get knowledge of mathematics literature.

i. To understand the importance of units of measurement.

Skill Objectives

A teacher of mathematics should help students acquire the following skill or ability objectives:

a. To develop the necessary skills to do numerical problems.

b. To develop the skill in preparing and using mathematical & statistical tables, ready recokners, and graphs.

c. To develop the skill of precision, brevity, accuracy, and neatness in oral as well as paper-and-pencil computations.

d. To develop the skill to review and check results.

e. To develop the ability to generalize concepts.

f. To develop the skill of making estimation.

g. To develop skill of posing and solving problems.

h. To develop the essential skill in drawing geometrical figures.

Appreciation Objectives

A teacher of mathematics should help students acquire the following appreciation objectives:

a. To appreciate the cultural values of the subject.

b. To appreciate the bread and butter value of the mathematics.

c. To appreciate the role of mathematics in the advancement of civilization.

d. To develop the appreciation of mathematics in developing cognitive abilities of the people.

e. To develop the appreciation of mathematics in making physical sciences more exact.

f. To develop the appreciation of mathematics in making analysis in economics and commerce.

g. To appreciate the social and cultural values of mathematics.

Attitude Objectives

Attitude toward mathematics is significantly related with achievement in mathematics. Sharma (2009) defined attitude towards mathematics as a learned predisposition to behave in a consistent evaluative manner toward the Mathematics. A teacher of mathematics should help students acquire the favourable attitude towards mathematics. The following attitude objectives can help students succeed in mathematics:

a. To develop the power of concentration and persistence among students.

b. To develop the attitude of self-study and verification.

c. To develop the attitude of mathematisation.

d. To enable the students to pose and solve open-ended questions.

e. To develop heuristic and discovery attitude among students.

f. To develop the attitude of preciseness, neatness, and exactness.

g. To develop habit of logical thinking and expression.

h. To develop the personal qualities of regularity, punctuality, unambiguousness, and truthfulness.

i. To help the students to be free from any kind of mathematics anxiety.

j. To develop in child the qualities of curiosity in mathematics and self-efficacy in mathematics.

k. To develop in child the habit of reading mathematics journals and magazines.

l. To help child participate in mathematics quizzes and competitions.

m. To inculcate the habit of solving mathematics puzzles and riddles.

Application Objectives

Mathematics has widest application in our lives. The students need to know following applications of mathematics:

a. To be able to apply school mathematics in solving daily life problems.

b. To able to use mathematical concepts and processes in others subjects efficiently.

c. To be able to know application of mathematics in all the aspects of our lives.

d. To prepare the students for higher mathematical studies.

e. To prepare the students for problem solving and mathematical games.

Check Your Progress

Notes :

(a) Write your answers in the space given below.

(b) Compare your answers with those given at the end of the unit.

1. What is major goal of mathematics education?

....

....

2. The true end of mathematics teaching is power and not knowledge. Give two reasons.

....

....

....

....

3. Mathematics has no recreation value. True/False

....

4.4 ANDERSON'S REVISED BLOOM'S TAXONOMY OF INSTRUCTIONAL OBJECTIVES

By taxonomy we mean an arrangement of ideas or a way to group things together. Anderson's revised Bloom's taxonomy of instructional objectives was developed by Anderson, Krathwohl, et. al. in a year 2000. This is a revised version of the original taxonomy developed by Bloom, Engelhart, Furst, Hill and Krathwohl (1956).

The original taxonomy had three dimensions – cognitive, affective, and psychomotor.

Cognitive Dimension: The cognitive domain had following categories-

Level	**Description**
1. Knowledge	**Remember previously learned information -** Recognize facts, terms, and principles
2. Comprehension	**Demonstrate an understanding of the facts. -** Explain or summarize in one's own words
3. Application	**Apply knowledge to actual situations -** Relate previously learned material to new situations
4. Analysis	**Break down objects or ideas into simpler parts and find evidence to support generalizations -** Understand organizational structure of material; draw comparisons and relationships between elements
5. Synthesis	**Compile component ideas into a new whole or propose alternative solutions -** Combine elements to form a new original entity
6. Evaluation	**Make and defend judgments based on internal evidence or external criteria** Make judgments about the extent to which material satisfies criteria

The action verbs that related with each level can help in formulation of instructional objectives. The action verbs for each level are given below:

Knowledge

List, define, tell, identify, show, label, collect, name, order, recognize, recall, repeat, state, quote, name, who, when, where.

Comprehension

Summarize, describe, interpret, contrast, predict, associate, distinguish, estimate, differentiate, discuss, extend, classify, discuss, explain, express, identify, indicate, locate, report, restate, review, select, translate.

Application

Apply, demonstrate, calculate, complete, illustrate, show, solve, examine, modify, relate, change, classify, experiment, discover, choose, dramatize, employ, illustrate, interpret, operate, practice, schedule, sketch, solve, use.

Analysis

Analyze, appraise, breakdown, calculate, categorize, compare, contrast, criticize, diagram, differentiate, discriminate, examine, experiment.

Synthesis

Integrate, modify, substitute, design, create, What if..., formulate, generalize, prepare.

Evaluation

Assess, rank, test, explain, discriminate, support, predict.

Affective Dimension: In the following the categories of affective domain along with the action verbs has been listed.

1. **Receiving** – willing to receive or reject new information accept, attend, develop, realize, receive, recognize, reply.
2. **Responding** – behave, complete, comply, cooperate, discuss, examine, obey, observe, respond.

3. **Valuing** – accept, balance, believe, defend, devote, influence, prefer, pursue, seek, value
4. **Organization** – codify, discriminate, display, favor, judge, order, organize, relate, systematize, weigh
5. **Characterization** – internalize, verify (formal instruction does not address)

Psychomotor Dimension: In the following the categories of psychomotor domain has been listed.

1. **Perception** – Using sense organs to obtain cues needed to guide motor activity.
2. **Set** – Being ready to perform a particular action
3. **Guided Response** – Performing under the guidance of a model
4. **Mechanism** – Being able to perform a task habitually with some degree of confidence and proficiency

These categories were according the psychological principles of learning, namely, simple to complex and concrete to abstract. In order to achieve the higher objectives like synthesis and evaluation, it was necessary that a student had mastery on the previous ones. This taxonomy has been widely used to classify objectives and test items. Some modifications were made in the original taxonomy of Bloom. The revised Bloom's taxonomy redefines the cognitive domain as the intersection of the Cognitive Process Dimension and the Knowledge Dimension.

4.4.1 Specifications

The revised taxonomy is a two-dimensional model. The **knowledge dimension**, in revised taxonomy, contains four main categories. In the revised taxonomy, the Metacognitive Knowledge category has been added. According to the developers, the Metacognitive Knowledge Category, "in some respects bridges the cognitive and affective domains". The specification of four categories of revised taxonomy is given below:

Factual Knowledge	Conceptual Knowledge	Procedural Knowledge	Metacognitive Knowledge
The basic elements that students must know to be acquainted with a discipline or solve problems in it.	The interrelationships among the basic elements within a larger structure that enable them to function together.	How to do something; methods of inquiry, and criteria for using skills, algorithms, techniques, and methods.	Knowledge of cognition in general as well as awareness and knowledge of one's own cognition.
• Knowledge of terminology • Knowledge of specific details and elements	• Knowledge of classifications and categories • Knowledge of principles and generalizations • Knowledge of theories, models, and structures	• Knowledge of subject-specific skills and algorithms • Knowledge of subject-specific techniques and methods • Knowledge of criteria for determining when • to use appropriate procedures	• Strategic knowledge • Knowledge about cognitive tasks, including • appropriate contextual and conditional • knowledge • Self-knowledge

The cognitive process dimension represents a continuum of increasing cognitive complexity—from remember to create. The specifications are as follows:

1. Remember – Retrieving relevant knowledge from long-term memory.
2. Understand – Determining the meaning of instructional messages, including oral, written, and graphic communication.
3. Apply – Carrying out or using a procedure in a given situation.
4. Analyze – Breaking material into its constituent parts and detecting how the parts relate to one another and to an overall structure or purpose.
5. Evaluate – Making judgments based on criteria and standards.
6. Create – Putting elements together to form a novel, coherent whole or make an original product.

4.4.2 Task Analysis

Concept of task analysis was popularized by Ryle. The task analysis focuses on the task to be performed. That is to say, it focuses upon the application aspect of the task. Task analysis is the analysis of how a task is accomplished. The task analysis is a detailed description of both manual and mental activities, task and element durations, task frequency, task complexity, environmental conditions and any other unique factors involved in performing a task. According Jonassen, Tessmer, & Hannum (1999), "Task analysis for instructional design is a process of analyzing and articulating the kind of learning that you expect the learners to know how to perform"

The task analysis of Anderson's revised bloom's taxonomy of instructional objectives may help in:

- ✓ Determining the instructional goals and objectives.
- ✓ Describing in detail the tasks and sub-tasks that the student will perform.
- ✓ Selecting learning outcomes that are appropriate for instructional development.
- ✓ Constructing performance assessments and evaluation.

The task analysis of Anderson's revised bloom's taxonomy of instructional objectives is given the following:

Knowledge Dimesion	Lower order thinking ——————→ Higher order thinking					
	Remember	Understand	Apply	Analyze	Evaluate	Create
A. Factual Knowledge ***B. Conceptual Knowledge*** ***C. Procedural Knowledge*** ***D. Metacognitive Knowledge***	Retrieving relevant knowledge from long-term memory.	Determining the meaning of instructional messages, including oral, written, and graphic communication.	Carrying out or using a procedure in a given situation.	Breaking material into its constituent parts and detecting how the parts relate to one another and to an overall structure or purpose.	Making judgments based on criteria and standards.	Putting elements together to form a novel, coherent whole or make an original product.
	Recognizing Recalling	***Interpreting Exemplifying Classifying Summarizing Inferring Comparing Explaining***	***Executing Implementing***	***Differentiating Organizing Attributing***	***Checking Critiquing***	***Generating Planning Producing***
	Remember Recognize Identify Recall Retrieve	Understand Interpret Clarify Paraphrase Illustrate Classify Categorize Summarize Generalize Infer Conclude	Apply Execute Carry out Use Implement	Differentiate Analyze Discriminate Focus Distinguish Select Organize Outline Integrate Structure Attribute Deconstruct	Evaluate Check Coordinate Detect Monitor Test Critique Judge	Create Generate Hypothesize Plan Design Produce Construct

As per Krathwohl (2002), the Taxonomy of Educational Objectives is a scheme for classifying educational goals, objectives, and, most recently, standards. It provides an organizational structure that gives a commonly understood meaning to objectives classified in one of its categories, thereby enhancing communication. The revised taxonomy has opened new possibilities for teachers and students and it is high time that we should update our self accordingly.

Check Your Progress

Notes :

(a) Write your answers in the space given below.

(b) Compare your answers with those given at the end of the unit.

4. Write major categories of Anderson's revised bloom's taxonomy of instructional objectives.

....

....

....

5. What are advantages of Task Analysis?

....

....

....

4.5 OBJECTIVES OF TEACHING ARITHMETIC, ALGEBRA, GEOMETRY

The objectives of teaching Arithmetic, Algebra, and Geometry can assist a teacher in planning, presenting and evaluating a lesson precisely. The role of teacher is to help students understand, apply and analyze different concepts of Arithmetic, Algebra, and Geometry. The objectives of teaching Arithmetic, Algebra, and Geometry are given here.

4.5.1 Objectives of Teaching Arithmetic

i. To enable the child to acquire the knowledge of facts and processes of arithmetic.

ii. To enable the child to acquire the knowledge of numbers and digits.

iii. To help the child to know about four fundamental operations of mathematics that is addition, subtraction, multiplication and division.

iv. To enable the child to know about whole numbers and integers.

v. To help the child understand fundamental theorem of arithmetic.

vi. To enable the child to get knowledge of fractions, decimals, exponents, scientific notations, percents, integers, proportions and word problems.

vii. To develop in the child ability to solve problems in arithmetic.

viii. To enable the students to solve modern, practical, every-day problems, of a simple nature.

ix. To enable the child to do calculations while shopping in the market.

x. To help the child to solve problems related to four fundamental operations of mathematics that is addition, subtraction, multiplication and division.

xi. To help the students solve simple and complex problems involving whole numbers, integers, decimals, exponents, scientific notations, percents, integers, proportions and word problems.

xii. To enable children to handle and understand the arithmetic under lying modern day-to-day life usage.

xiii. To develop the ability to verify all results.

xiv. To develop in students ability to solve open-ended and divergent problems in arithmetic.

xv. To help the child to do accurate calculations at a faster speed.

xvi. To develop in the child the cultural values of arithmetic.

xvii. To give child training in generalizing so that he/she develops attitude to generalize the concepts.

xviii. To enable the child to become a useful and desirable citizen.

xix. To develop in the child attitude to function as a mathematician.

xx. To develop the attitude to check all results.

xxi. To develop in child favourable attitude towards arithmetic.

xxii. To develop in the child capacity and habit to mathematise.

4.5.2 Objectives of Teaching Algebra

i. To enable the child know about Laws of indices -

a) $a^0 = 1$

b) $a^{-m} = \frac{1}{a^m}$

c) $a^m \times a^n = a^{m+n}$

d) $a^m \div a^n = a^{m-n}$

e) $(a^m)^n = a^{mn}$

f) $a^{m/n} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$

ii. To enable the child to recall, verify and application of algebraic identities in factorizations of polynomials such as

a) $ax + bx + cx = x(a + b + c)$

b) $(a + b)^2 = a^2 + 2ab + b^2$

c) $(a - b)^2 = a^2 - 2ab + b^2$

d) $a^2 - b^2 = (a + b)(a - b)$

e) $(x + a)(x + b) = x^2 + (a + b)x + ab$

f) $(a + b)^3 = a^3 + b^3 + 3ab(a+b)$

g) $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

h) $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

iii. To develop in the child understanding of constant, variables and functions.

iv. To develop in the child knowledge about use of functions in other fields like science, economics, and management.

v. To enable the child to understand linear, simultaneous, and quadratic equations.

vi. To develop the ability to solve formal problems in variation and proportion

vii. To develop the ability to solve problems from real life, including problems on Ratio and Proportion and with algebraic and graphical solutions being done simultaneously.

viii. To enable the child to solve linear, simultaneous, and quadratic equations by graph.

ix. To help child understand arithmetic progressions and their application in solving daily life problems.

x. To develop the ability to review and check results.

4.5.3 Objectives of Teaching Geometry

i. To provide training in logical thinking.

ii. To develop space intuition among students.

iii. To enable the child make practical use the essential formulas of mensuration.

iv. To develop in student ability to discover theorems.

v. To help child to detect and appreciate geometric forms in day-to-day life.

vi. To foster deductive reasoning among students.

vii. To help child understand the role of geometry in civilization.

viii. To develop the ability of spatial imagination.

ix. To enable the child to understand facts, properties, and principles of geometry.

x. To help the child acquire ability to measure with ruler and compasses.

xi. To develop in child the ability to make constructions.

xii. To develop in the child the power of original, logical, geometrical reasoning.

Here it would be pertinent to mention that according to Van Hieles, there are five levels of thinking in geometry. The teachers can consider these levels as five objectives of teaching geometry. A student can achieve one level of thinking only after he/she has passed through the previous one. **The Van Hieles Levels of Thinking in Geometry are as follows:**

Level 0 : The student identifies, names, compares and operates on geometric figures (for example, triangles, angles, intersecting or parallel lines) accoding to their appearance.

Level 1 : The student analyzes figures in terms of their components and relationships among components and discovers properties/rules of a class of shapes empirically (for example, by folding, measuring, using grid or diagram)

Level 2 : The student logically interrelates previously discovered properties/ rules by giving or following informal arguments.

Level 3 : The student provides theorems deductively and establishes interrelationships among network of theorems.

Level 4 : The student establishes theorems in different postulational systems and analyzes/compares these systems.

The objectives of teaching arithmetic, algebra, and geometry given above can provide guidance to the mathematics teacher for planning, executing and evaluating mathematics learning of the school students.

Check Your Progress

Notes :

(a) Write your answers in the space given below.

(b) Compare your answers with those given at the end of the unit.

6. Teaching of Geometry helps in developing spatial ability of the students. True/False

   ....

7. To help the students solve simple and complex problems involving whole numbers, integers, decimals, exponents, scientific notations, percents, integers, proportions and word problems

   The above stated objective is for Teaching of ....

8. Van Hiele gave .... level(s) of thinking in geometry.

4.6 LET US SUM UP

The major goal of mathematics education is to develop the abilities of students to mathematise. Apart from bread and butter value, teaching of mathematics also has Disciplinary, Utilitarian, Cultural, Social, and Recreational values. Bloom has given taxonomy of objectives for the formulation of behavioural objectives. Anderson's revised Bloom's taxonomy of instructional objectives in a year 2000. The revised taxonomy allows teachers to analyze and categorize students' tasks. Moreover, the revised taxonomy helps in alignment of planning, presentation and evaluations.

4.7 LESSON–END EXERCISE

1. Discuss the objectives of teaching mathematics at school stage.
2. What are instructional objectives? Prepare Knowledge, Understanding and Application objectives on the topic of your choice.
3. Describe Anderson's revised Bloom's taxonomy with reference to specifications.

4. Discuss task analysis in relation to Anderson's revised Bloom's taxonomy.

5. List objectives of teaching arithmetic, algebra and geometry.

4.8 SUGGESTED FURTHER READINGS

Fuys, D., Geddes, D., & Tischler, R. (1988). The van Hiele model of thinking in geometry among adolescents [Monograph]. *Journal for Research in Mathematics Education*, 3, 1-196.

NCF (2005). *National Curriculum Framework*. New Delhi: National Council of Educational Research and Training.

Kumar, S. (1997). Teaching of mathematics. New Delhi: Anmol Publications Pvt. Ltd.

Gakhar, S. C. (2006). Teaching of mathematics. Panipat: N.M. Publication

Anderson, L. W. et. al. (2001). A taxonomy for Learning, teaching, assessing: a revision of bloom's taxonomy of educational objectives. New York: Longman. Available at *file:///E:/jammu/jammu/lesson%204/Anderson-Krathwohl%20-%20A%20taxonomy%20for%20learning%20teaching%20and%20assessing.pdf*

4.9 ANSWERS TO CHECK YOUR PROGRESS

1. According to The National Curriculum Framework (NCF) (2005) the major goal of mathematics education is developing children's abilities for mathematisation. Mathematics aims to develop reasoning ability of the students and does not give much emphasis on cramming.

2. Mathematics stimulates aims at developing thinking abilities that helps in applying the acquired knowledge.

3. False, the mathematical recreations are as old as mathematics itself

4. The categories are as follows:

   a. Remember – Retrieving relevant knowledge from long-term memory.

b. Understand – Determining the meaning of instructional messages, including oral, written, and graphic communication.

c. Apply – Carrying out or using a procedure in a given situation.

d. Analyze – Breaking material into its constituent parts and detecting how the parts relate to one another and to an overall structure or purpose.

e. Evaluate – Making judgments based on criteria and standards.

f. Create – Putting elements together to form a novel, coherent whole or make an original product.

5. The advatages of task analysis of Anderson's revised bloom's taxonomy of instructional objectives are as follows:

   a. Determining the instructional goals and objectives.

   b. Describing in detail the tasks and sub-tasks that the student will perform.

   c. Selecting learning outcomes that are appropriate for instructional development.

   d. Constructing performance assessments and evaluation.

6. True

7. Arithmetic

8. Five

—×—

LESSON NO. 5 **UNIT-II**

APPLICATION OF APPROACHES AND MODELS OF TEACHING MATHEMATICS

STRUCTURE

5.1 INTRODUCTION

Although technology has entered into the every nook and corner of a classroom, the methods and approaches adopted by a teacher for promoting students' learning still holds the key. In the present times, the role of a teacher in a classroom has been changed from an authoritarian one to a facilitator one. The methods and approaches used by a teacher to facilitate the learning of students havea significant importance.Approaches and models of teaching–learning mathematics are the systematic procedures of presentation of content in the classroom. Effective learning and management of a classroom depend upon the right approaches used by the teacher.The approaches and models of teaching mathematics to be discussed in this chapter may provide necessary guidance to teacher trainees as well as mathematics teachersforeffective classroom practices in promoting mathematics learning.

5.2 OBJECTIVES

After you study this lesson, you will be able to:

1. Describe inductive – deductive approach of teaching mathematics.
2. Write merits and demerits of inductive – deductive approach of teaching mathematics
3. Give examples of inductive – deductive approach of teaching mathematics.
4. Explain analytic - synthetic approach of teaching mathematics.
5. Discuss merits and demerits of analytic - synthetic approach of teaching mathematics
6. Give examples of analytic - synthetic approach of teaching mathematics.
7. Describe the guided discovery approachof teaching mathematics.
8. Discussproject method of teaching mathematics.
9. Give examples of project method of teaching mathematics.
10. Explain concept attainment model of teaching mathematics.

5.3 APPLICATION OF APPROACHES AND MODELS OF TEACHING MATHEMATICS

5.3.1 Inductive – Deductive Approach

The inductive – deductive approach is a combination of two different but complimentary approaches, namely, Inductive Approach and Deductive Approach. Here we will try to understand these approaches.

INDUCTIVE APPROACH

The inductive approach is one of the oldest methods of instruction. The inductive approach is about creating scientific knowledge. As a student of mathematics you must have proved many formulae or theorems by using induction technique. In induction we prove a formula by showing that if a formula is true for a particular case, it is true for all the cases.The inductive approach has been based upon the inductive reasoning. The inductive reasoning says that if a statement is true for a number of cases it will be true for all the cases. The inductive approach proceeds from -

Particular to General

Known to Unknown,

Concrete to Abstract,

Example to Formula, or

Hypothesis to Conclusion

Use of inductive approach in a mathematics class

Example 1: To prove that sum of angles of a quadrilateral is 360^0.

Step 1. Ask all the students in a class (say 20) to draw a quadrilateral ABCD in their notebooks.

Step 2. Ask students to measure angles A, B, C and D of the quadrilateral ABCD they have made in their notebooks.

Step 3. Prepare a table on the board as shown below and ask all the students one-by-one to tell the measure of angles of quadrilateral ABCD they have made in their notebooks.

Step 4. Write sum of angles of a quadrilateral in each case.

Step 5. Ask them what they can conclude?

The students may conclude that **Sum of Angles of a Quadrilateral is 360^0.**

Sr. No.	Angle A	Angle B	Angle C	Angle D	Sum of Angles of a Quadrilateral ABCD

Example 2 : To verify the identity $a^2 - b^2 = (a - b)(a + b)$.

Step 1. Suppose there are 20 students in a class. Divide 20 students into a 5 groups.

Step 2. Give each group a task of multiplying (a – b) and (a + b) using different alphabets, like Group 1 will multiply (p – q) with (p + q); Group 2 will multiply (l – m) with (l + m); Group 3 will multiply (r – s) with (r + s); Group 4 will multiply (y – z) with (y + z); and Group 5 will multiply (e – f) with (e + f);

Step 3. Prepare a table on the board as shown below and ask all the groups one-by-one to tell their multiplication terms,product and result they have got after multiplication.

Step 4. Ask them what they can conclude?

They may conclude that first term^2 – second term^2 = (first term – second term) (first term + second term) or $a^2 - b^2 = (a - b)(a + b)$.

Group	Multiplication Terms	Product	Result
1	(p – q) and (p + q)	$p^2 - q^2$	$(p - q)(p + q) = p^2 - q^2$
2	(l – m) and (l + m)	$l^2 - m^2$	$(l - m)(l + m) = l^2 - m^2$
3	(r – s) and (r + s)	$r^2 - s^2$	$(r - s)(r + s) = r^2 - s^2$
4	(y – z) and (y + z)	$y^2 - z^2$	$(y - z)(y + z) = y^2 - z^2$
5	(e – f) and (e + f)	$e^2 - f^2$	$(e - f)(e + f) = e^2 - f^2$

MERITS OF INDUCTIVE METHOD

1. Scientific Method

   As in scientific method, while applying inductive approach we follow a proper sequence and do not believe in hearsay.

2. Discovery Approach

   In inductive method formula is not dictated by the teacher but rather discovered by the students.

3. Reasoning Ability

   The inductive method gives students training in reasoning ability. Question of 'How' and 'Why' asked by teacher during the inductive approach helps in the development of reasoning ability.

4. Discourages Rote Leaning

   In inductive approach students have no stress of cramming the formula. Students discover the formula and undue rote learning has no place in inductive approach.

5. Active Leaning

   The discovery of formula with inductive approach is not possible if students are passive or inattentive. The inductive approach encourages active participation of the students in mathematics learning.

6. Based on Psychological Principals

   Mathematics learning with inductive approach is based upon psychological principals of Learning by Doing and Concrete to Abstract.

7. Based on Maxims of Teaching

   The inductive approach follows maxims of teaching like Seen to Unseen, Known to Unknown and Definite to Indefinite.

8. Good to Start With

   Most of the mathematics teaching is inductive in the beginning. Also this gives opportunity to create good rapport between teacher and students. So, it is one of the best suitable methods for young children.

9. No Place for Boredom

   As young children are involved different activities while learning mathematics with inductive approach, they find this method as interesting and satisfying.

10. Self-Study

    The students themselves have to think examples, find similarities and dissimilarities among examples, and make generalization so they think they are the master of their learning. This encourages habit of self-study in the students.

DEMERITS OF INDUCTIVE METHOD

1. Not Suitable for Higher Classes

   In higher classes students can get bore when they are taught mathematics with this method. Also, it may not be possible to cover the entire syllabus at higher level with inductive approach. So, teaching mathematics with inductive approach is not very useful at higher level.

2. Too Much Expectations

   While teaching with inductive approach a teacher expects that students are able to think various examples and then reason out generalization, this may be equivalent to asking too much from young children

3. More Suitable for Bright Students

   The reasoning ability of all the students is not same. Although some gifted ones may be able to derive formula with this inductive approach, many others may be at the receiving end.

4. Excessive Workload

   The method demands lot of preparation and implementation in terms time and labour. This may increase the workload of a teacher already overburdened with many academic as well as non-academic activities.

5. Not Conclusive

   The inductive reasoning employed while teaching with inductive approach may not be conclusive due insufficient examples presented. Moreover, mathematics at advanced stage is deductive in nature and only inductive approach is not sufficient in teaching mathematics concepts satisfactorily. The topics covered with inductive approach need to be practiced enough.

DEDUCTIVE APPROACH

The opposite of inductive approach is deductive approach.The deductive reasoning used in deductive approach is essential part of mathematics.In deductive approach, particular case(s) are inferred from a theory or formula. As opposite to inductive approach, in deductive approach, if a thing is true in general, it is also true for all the particular cases. We hold a theory or generalization as truth and deduce particular truths from that. The deductive approach proceeds from -

General to Particular,
Unknown to Known,
Abstract to Concrete,
Formula to Example, or
Conclusion to Hypothesis.

Use of deductive approach in a mathematics class

In deductive approach first a formula is given, then some examples, and then practice is given.

Example 1 : Surface Area of a Sphere.

Step 1. Teacher will say, "Dear Students! Today we are going to learn Surface Area of a Sphere."

Step 2. Teacher will tell that formula for finding Surface Area of a Sphere is $4 \Pi r^2$.

Step 3. Teacher will solve some problems on the board in which formula for Surface Area of a Sphere is used.

Step 4. Teacher will ask students to solve similar problems.

Example 2: $a^2 - b^2 = (a - b)(a + b)$.

Step 1. Teacher will say, "Dear Students! Today we are going to learn that $a^2 - b^2 = (a - b)(a + b)$."

Step 2. Teacher will tell that $a^2 - b^2 = (a - b)(a + b)$.

Step 3. Teacher will solve some problems that can be solved with the help of identity$a^2 - b^2 = (a - b)(a + b)$.

Step 4. Teacher will ask students to solve similar problems.

MERITS OF INDUCTIVE METHOD

1. Effective Drill

   Deductive approach can be used effectively for drill work as this approach provides thorough practice and revision of the concepts and formulae.

2. Saves Time and Energy

   The deductive approach saves time and energy of the teachers and students and lots of problems can be solved in a short time by providing already established formula.

3. Complementary to Inductive Approach

   As deductive approach eliminates the inadequacy of inductive approach, it is said to be complementary to inductive approach.

4. Suitable for Masses

   With deductive approach all students can work on a same pace and had advantage for all type of students, so this approach is suitable for all the students in the class.

5. Advantageous for Higher Classes

   As no time is wasted in establishing formula in deductive approach, this approach is beneficial for teaching mathematics to higher classes.

6. Speed and Efficiency

   The deductive approach helps is solving many problems in short time without putting much burden on the students and teachers, so this approach is suitable for thedevelopment of speed and efficiency in doing mathematics.

7. Covers All Topics

   With deductive approach maximum syllabus can be covered in minimum time. The approach is useful for all the topics of mathematics.

8. Training for Memory

   Memory is one of our important cognitive ability. While learning mathematics with deductive approach students have to memorize many formulas, laws and concepts, thereby improving memorization ability of the students.

9. Tackles Indiscipline

   The deductive approach is a teacher-centered approach and helps in minimizing the incidents of indiscipline in the large classrooms.

10. Suitable for Traditional Learners

    The approach is suitable for students who do not have enough experience of modern approaches of discovering formula themselves.

DEMERITS OF INDUCTIVE METHOD

1. Un-Psychological Approach

   The deductive approach is not child-centered as emphasis is on the attainment of facts and formularather than teaching in accordance with the principles of psychological teaching.

2. Encourages Rote-Learning

   The deductive approach encourages rote-learning as much emphasis is on cramming formulas and rules without actually knowing how they have been established. The cramming is glorified over understanding.

3. Passive Learning

   Due to lack of democratic nature of the deductive approach, the students are not very much interested in the mathematics learning. The deductive approach makes students passive and inattentive learners.

4. Not Suitable for Elementary Classes

   The elementary school students find it difficult from where an abstract formula has erupted. The deductive approach is not suitable for beginners and they may start fearing abstract mathematics concepts.

5. No Training for Thinking and Reasoning

   The deductive approach provided no training to the important cognitive abilities like thinking and reasoning necessary for surviving and succeeding in a real life.

6. Stress for Mind

   The deductive approach can put unnecessary burden on the mind of remembering and recalling a formula without understanding it. The stress of remembering and recalling a formula without understanding it can result in mathematics anxiety.

The discussion related with inductive and deductiveapproaches can be concluded by saying that both inductive and deductiveapproaches are complimentary to each other. In order to have proper learning of mathematics, students should be taught with inductive approach at the beginning and then for the application and revision of formula deductive approach needs to be used.

5.3.2 Analytic – Synthetic Approach

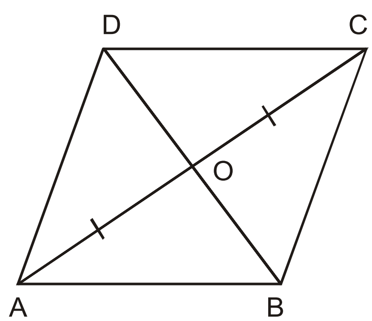
The inductive – deductive approach is a combination of two different but complimentary approaches, namely, Analytic Approach and Synthetic Approach. Here we will try to understand these approaches.

ANALYTICAPPROACH

The analyticapproachhas been derivedfrom analysis. Analysis is breaking a problem into its constituent parts in order to understand or find solution of it. In analytic approach we start from unknown part of the problem, break that unknown into simpler parts, and then with the help of reasoningreach atsolution. The unknown part is connected with known part with analytic reasoning to get the result. The analytic approach proceeds from Unknown to Knownor To Prove to Given.

Example 2 : Show that the diagonals of a rhombus are perpendicular to each other.

Solution.



Teacher : What is given to us ?

Student 1 : A rhambus ABCD. AC and BD are its two diagonals, intersecting each other at O.

Teacher : What is to be proved?

Student 2 : $\angle AOD = 90°$ or $\angle AOB = 90°$

Teacher : How we can prove this ?

Student 3 : If we can show that $\angle AOD = \angle COD$

Teacher : How we can get this ?

Student 4	:	By proving triangles congruent.
Teacher	:	Which triangles we need to prove congruent ?
Students 5	:	ΔAOD and ΔCOD
Teacher	:	Which congruence rule will help us ?
Student 6	:	SSS congruence rule
Teacher	:	Which rides are equal, respectively ?
Student 7	:	$OA = OC$ $OD = OD$ $AD = CD$
Teacher	:	Why $OA = OC$?
Student 8	:	Diagonals of a parallelogram bisect each other.
Teacher	:	As triangles are now congruent, so what we get ?
Student 9	:	$\angle AOD = \angle COD$
Teacher	:	Why ?
Student 10	:	Corresponding parts of congruent triangles are congruent (CPCT).
Teacher	:	What is $\angle AOD = \angle COD = ?$
Student 11	:	$180°$
Teacher	:	Why ?
Student 12	:	Linear pair
Teacher	:	So what we get ?
Student 13	:	$\angle AOD = 90°$
Teacher	:	What do you conclude then ?
Students 14	:	The diagonals of a rhombus are perpendicular to each other.

SYNTHETIC APPROACH

The syntheticapproachis opposite to analytic approach. Synthetic approach has roots in synthesis. Synthesis is joiningtogether the parts in order to understand or find solution of a problem. Here we start from known part of the problem and by combining parts reach at the unknown part. Theknown part is connected with unknown part to get the required result. The synthetic approach proceeds from Known to Unknown or Given to To Prove.

Example 1 : If $\frac{p}{q}=\frac{r}{s}$, prove that $\frac{pr-2q^2}{q}=\frac{r^2-2qs}{s}$

Solution : We have $\frac{p}{q}=\frac{r}{s}$

Substract $\frac{2q}{r}$ from both sides

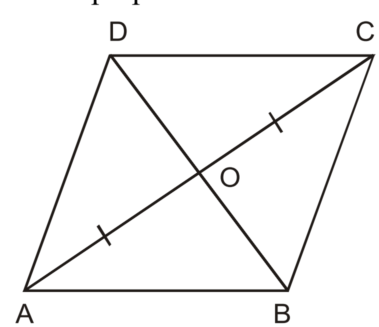
$\therefore \quad \frac{p}{q}-\frac{2q}{r}=\frac{r}{s}-\frac{2q}{r}$ (No logic or reason why $\frac{2q}{r}$ has been subtracted)

or $\quad \frac{pr-2q^2}{qr}=\frac{r^2-2qs}{sr}$

or $\quad \frac{pr-2q^2}{q}=\frac{r^2-2qs}{s}$

Hence the result.

Example 2 : Show that the diagonals of a rhambus are perpendicular to each other.



Solution. Consider the rhambus ABCD.

Here AB = BC = CD = OA

[Sides of rhambus are equal]

In ΔAOD and ΔCOD

OA = OC (Diagonals of a parallelogram bisect each other)

	OD = OC	(Common)
	AD = CD	
∴	$\Delta AOD \cong \Delta COD$	(SSS congruency)
∴	$\angle AOD \cong \angle COD$	(CPCT)
But	$\angle AOD + \angle COD = 180°$	(Linear pair)
So	$2\angle AOD = 180°$	
∴	$\angle AOD = 90°$	

Thus, the diagonals of a rhombus are perpendicular to each other.

CRITICAL ANALYSIS OF ANALYTIC – SYNTHETIC APPROACH

S.No.	ANALYTIC APPROACH	SYNTHETIC APPROACH
1.	It has roots in analysis. The problem is broken into simple parts.	It has been based upon synthesis.
2.	Analytic approach starts from unknown and ends at known.	Synthetic approach starts from known and ends at unknown.
3.	Analytic approach proceeds from what is to be proved and reaches at what is given.	Synthetic approach proceeds from what is given and reaches at what is to be proved.
4.	Analytic approach is based upon logic. It leaves no doubt in the mind of the leaner.	Synthetic approach is not logical. Many doubts remain unclear in the mind of the leaner.
5.	Analytic approach is based on inductive reasoning.	Synthetic approach is premised upon deductive reasoning.
6.	It promotes understanding.	It promotes rote-learning.
7.	It develops scientific attitude among students.	It does not promote scientific attitude among students.
8.	Analytic approach facilitates the reasoning and thinking cognitive components of mind.	Synthetic approach facilitates the memory ability of mind.
9.	Analytic approach is heuristic in nature.	Synthetic approach is not heuristic in nature.
10.	Discovery learning is the theme of analytic approach.	Synthetic approach is of a crammer .
11.	Analytic approach is lengthy, laborious and time consuming.	Synthetic approach is precise and short.
12.	Analytic approach gives no practice of attaining efficiency and speed.	Synthetic approach develops the efficiency and speed in solving problems.
13.	The Analytic approach is psychological one. The student participation is encouraged and the whole approach is democratic in nature.	It is un-psychological approach. The student participation is not encouraged and the whole approach is dominated by the teacher.
14.	Analytic approach is informal and solutions are not properly finished.	Synthetic approach is more formal and solutions are properly finished.
15.	In analytic approach each step has reason and facts if forgotten can be rediscovered.	In synthetic approach each step does not have reason and facts if forgotten cannot be rediscovered.

In the end, it may be said that both analytic and synthetic approaches are inter-dependent and complimentary to each other. Both approaches have own merits and demerits. Application of synthetic approachprior to analytic can be rigid and un-psychological butsynthesis after analysis is necessary for the follow-up work.

5.3.3 Guided Discovery Approach

Discovery learning can be traced from the word heuristic, which means to find out, discover. In educational settings, learning through exploration is sometimes called discovery learning. Discovery learning is an important component of modern constructivist approaches. In discovery learning students are encouraged to learn largely on their own through active involvement with concepts and principals, and teachers encourage students to have experiences and conduct experiments that permit them to discover principles for themselves.

Bruner (1966), an advocate of discovery learning, said, "we teach a subject not to produce littlie living libraries on that subject, but rather to get a student to think.....for himself, to consider matters as an historian does, to take part in the process of knowledge getting. Knowing is a process, not a product.

Piaget suggested that effective discovery learning should be largely a child-initiated andchild-directed effort. By and large, however, researchers are finding that students benefit morefrom carefully planned and structured activities that help them construct appropriate interpretations.

Advantages of Discovery Learning

1. It arouses students' curiosity motivating them to work until they find answers.
2. Students also learn independent problem-solving and critical-thinking skills, because they must analyze and manipulate information.
3. The students understand what they learn and are thus better able to remember it and to use it in other situations.
4. The students learn a way of going about learning on their own; in mathematics they learn some techniques, or strategies, for discovering other new things.

5. They develop interest in what they are learning.

Methods to Aid in Discovery Learning

The Inductive Method: The inductive method is one method which can be used to arrive at discoveries. It is, perhaps the major kind of thinking expected in activities in which the teacher is hoping to have the students reach a generalisation by seeing a pattern existing in a collection of data. By the inductive method we commonly think of procedures by which the study and analysis of the relationships in several but not all specific instances in some class leads to a generalisation about those relationships for all instances in the class.

The Deductive Method: Some teachers feel it is not possible to discover by thinking deductively. Perhaps this is because their own school experience with deduction has been almost entirely from textbooks in which the generalisation is stated first, and the student's job is to apply it or to prove it a generalisation (theorem) in some mathematical structure. But a student may, for example, reason that since the sum of the measures of the interior angles of a triangle is 180^o and since a quadrilateral can be divided into two triangles, then it is clear, without testing several instances, that the sum of measures of the interior angles of a quadrilateral is 360^o. That is, he discovers a property of quadrilaterals by deducing that it must follow from his knowledge about triangles.

Discovery by Analogy: Students can make considerable use of analogy in mathematics.For example, the locus of all points in a plane equidistant from a given point is a circle. By analogy, the locus of all points in a space equidistant from a given point is a sphere.Polya writes, "Analogy is a sort of similarity. Similar objects agree with each other in some respect, analogus objects in certain relations of their respective parts." He repeatedly suggests that the person confronted with a new problem should first search his repertoire of knowledge for problems he has solved previously and which are analogous to, or have many similar elements with, or are related to the new problem. Reasoning by analogy has pitfalls too. You have seen your students do such things as follows:

if $(x-2)(x+3)=0$, then $x-2=0$ or $x+3=0$. By analogy if $(x-2)(x+3)=5$, then $x-2=5$ or $x+3=5$.

Cautions to be taken while using discovery approach

1. Discovery learning requires a teacher to have much deeper understanding and mastery, in a pure mathematical sense, of the subject matter than lecturing does.
2. Teacher should recognize, as quickly as possible, the validity of unexpected responses.
3. Teacher should be able to tell when a response which is not correct as stated nevertheless includes a valid idea, so that the discussion can be guided in the direction of the valid idea.
4. The teacher must be mindful of relates to the fact that not all students make a sought for discovery, even when a generous amount of time is allowed. The teacher must guide learning process by using one of the following approaches:
   (i) Providing "answer-giving" instructions and withholding "answer-seeking" instructions: Answer-giving instructions on the part of teacher do not aid the learner to develop a power to discover principles. The students do not learn a strategy for identifying new principles. Such instructions apply to the specific problem the student is working with; they do not have the generality to help him learn a strategy for discovering.
   (ii) Providing "answer-seeking" instructions and withholding "answer-giving" instructions: By using this approach the teacher might ask the student to think out loud for a given problem, so that he can determine the student's approach. If the student has persisted in one approach and it is not "paying off," the teacher might suggest an alternative attack. Thus, he helps develop an important aspect in the process of discovering, flexibility.

The teacher who uses discovery learning approach needs a strong preparation in mathematics; but if he is successful with these methods, he should expect some student discoveries for which he does not have the background to determine on the spot their validity. He should be willing to learn along with his students, to do some research and discovering himself. Stephen Willoughby "———some of the potential good in the discovery method may be missed if the teacher knows precisely what he is looking for does not encourage the pupils to continue to think along lines that are unfamiliar to both pupils and teacher."

Check Your Progress

Notes :

(a) Write your answers in the space given below.

(b) Compare your answers with those given at the end of the unit.

1. Inductive method starts from ………………. and ends at …………........

2. What is synthetic approach in teaching mathematics?

   ....

   ....

3. Give methods of discovery in mathematics.

   ....

   ....

   ....

5.3.4 Project Method

The project method is a teacher-assisted collaborative approach in which students acquire and apply knowledge and skills to define and solve realistic mathematical problems.In project methodstudents solve a practical problem over a period of several days or weeks. The project method has roots in the philosophical disposition of John Dewey and American Progressive education movement. In 1918, William H. Kilpatrick popularized project method in education system with his article, The Project Method.Kilpatrick pointed out that project method as the purposeful act is the typical unit of the worthy life. Also, Stevenson said that a project is problematic act, carried to completion in its most natural setting.

Procedure of Project Method

The project method has five steps:

1) **Providing / creating the situation**

   The role of teacher is to help students finda problem or situation that can be solved with project method. Both convergent and divergent thinking can help in selection of ideas and the topic.

2) **Proposing and Choosing the Project**

   While proposing or choosing the project, it must be kept in mind that every topic cannot be taken for project. Also, teacher should guide the students so that they do not choose those projects which require little efforts or those which are beyond the scope of their grade.

3) **Planning the Project**

   The planning part of the include distribution of the tasks to be accomplished and the timetable of key events. Here we need to assign group leaders andduties need to be assigned to the students.Also, the teacher can pose questions like 'What methods are needed in order to get our target?' 'Who or what could be of use for us?' And 'Who or what obstructs us?' The deadline has to be decided for completing each step.

4) **Execution of the Project**

   The execution of the project means application of the strategy devised in the previous step. However, execution can hamper when there is absence of common understanding; students are not motivated or have become disinterested;uninspiringgroup leaders; vague goals and procedures; and improper monitoring and feedback of project.

5) **Evaluation of the Project**

   This phase demands systematic and objective assessment of an ongoing or completed project. The evaluation can be formative or summative one, which can done at four levels – self, peer, teacher, and parents. This should help in assessing how much learning has taken place; how far the objectives have been met; how much each member has contributed; and how it could have done better.

6) **Recording and Reporting of the Project**

The mathematics project undertaken by the students should be recorded and reported. Recording and reporting of project acts as evidence or a guideline. The recording and reporting allows people to retain project for future use.In the age of digital technology, the final reporting needs to be in the digital form. This will save wastage of paper and space.

Advantages of Project Method

1. The project methodhelps students to independently acquire mathematics knowledge and skills.
2. The project method helps in the integration of knowledge from various subject areas.
3. The project method develops better study habits.
4. The students develop attitude of cooperation and collaboration, which is necessary for surviving and succeeding in the modern society.
5. The project method makes students research oriented.
6. The project method helps in improving the habit of self-study.
7. The project method helps in fostering creativity among students.
8. It promotes learning by doing and dignity of labour.
9. The project method can develop self-efficacy in mathematics among students. It may also develop students' confidence in doing mathematics.
10. The learning happened with project method is durable and practical.
11. The project method develops democratic values among students.
12. The project develops the qualities of tolerance, sense of responsibility, persistence, resourcefulness, and self-discipline.

Disadvantages of Project Method

1. The project method demand quality mathematics teachers, which is no mean task. There is dearth of quality and motivated mathematics in India.
2. Mathematics teachers are already overburdened in the schools and it is very difficult for a teacher to plan or to execute the projects to the learners and supervise them.
3. As the syllabus of school mathematics is large, it would be impossible to finish syllabus in time with project method teaching. The project method is lengthy and time-consuming.
4. The brilliant students may become irritated when they find that the whole work has become only their responsibility and the other members are not putting any effort to complete the project.
5. The project method is costly from the point of view of time, energy, and cost.

Examples of Some Projects inMathematics

1. Celebration of National Mathematics Day.
2. Organization of mathematics trip.
3. Development of mathematics garden.
4. Economic and population survey.
5. Water and electricity consumption in the area.
6. School health and attendance record.
7. Survey of nearby school/institution/factory.
8. Construction/colour-wash of a building.
9. Rain and temperature record of the area.
10. Purchasing and selling commodities from market.
11. Running of cooperative mess.

Although it may not be possible for a mathematics teacher to cover all the topics of mathematics with project method, but keeping in mind the advantages project method has, some topics need to be taught withproject method.

5.3.4 Concept Attainment Model

A model of teaching is a plan or pattern that can be used to shape curriculum, long term cause of study, to design instructional materials, and to guide instruction in the class room and other settings (Joyce and Weil,1980). There is large variety of teaching models available and the Concept Attainment Model falls under the category of information processing models.The concept attainment model has been developed by Bruner, Goodnow and Austin(1967).The Bruner's concept attainments model states that the role of teacher is to create situation in which student can learn on their own rather than to provide packed information to students. The students learn the concept through concrete examples and non-examples. This model helps the students to develop and form new concept. The concept attainment model has been explained with the help of a lesson plan given in the following:

Lesson Plan in Accordance withConcept Attainment Model

Subject – Mathematics **Class** – VIII

Topic – Prime Numbers **Time** – 30 min

Curriculum Objectives –

1. To enhance the critical thinking, reasoning power of students.
2. To develop interest of students in mathematics.

Specific Objectives – After the lesson, the students will be able —

1. To define Prime numbers.
2. To tell essential attributes of prime numbers.
3. To tell the examples of prime numbers.
4. To differentiate between prime numbers and other numbers.
5. To recognize the examples of prime numbers.

Prior Knowledge Needed – It is assumed that students are familiar with the term factors or multiples etc.

Material required – A chart showing list of examples/non-examples related to topic.

Procedure

Phase–I— Presentation of Data and Identification of Concept

— Teacher will present labeled examples.

— The pupils are told that there is only one idea common in all the +ve examples and that they have to compare and justify the attributes and form some hypothesis about the concept.

Examples of the Concept
&
Students' hypothesis

Sr. no.	Positive examples (yes)	Negative examples (no)	Students' hypothesis
1.	3	8	Number which is divisible by 3
2.	5	-5	Natural no.
3.	7	12	One-digit natural no.
4.	11	6	Odd natural no.
5.	13	-13	Odd natural no.
6.	19	21	Odd natural number, less than 20
7.	2	9	Natural no.s less than 20
8.	23	25	Natural no.s which are divisible by itself.
9.	29	1	Natural no.s which are divisible by 1 and the no. itself

— Students state definition according to the essential attributes as— "Natural numbers which are divisible by 1 and the no. itself."

Phase-II— Testing attainment of the concept

— Teacher will present some unlabeled examples.

— Students will identify additional unlabeled examples as 'yes' or 'no'.

Examples	Students' hypothesis
7	Yes
3	Yes
19	Yes
1	No
2	Yes
11	Yes

— Now, the teacher will confirm students' hypothesis.

— Teacher will name the concept as—

"Prime Numbers"

— Teacher will restate definition according to essential attributes as—

"The numbers which has only two factors namely 1 and the number itself, are known as Prime Numbers."

— Student will generate examples at their own.

Phase-III— **Analysis of thinking strategies / Evaluation**

1. What came in your mind when first few examples were presented to you?
2. What changes occurred in your guesses when few more examples were given to you?
3. Do you think more examples should have been presented to you?
4. Define Prime Numbers.
5. 1 is not a prime number. why?

   Give reasons.
6. Differentiate b/w following numbers—

   7, 2, 19, 15, 6, 17, 14, 35

Advantages of Concept Attainment Model (CAM)

1. CAM develops cooperation among students as they work together in cooperative groups to attain concepts.
2. The students become familiar with strategies of hypothesis formulation and testing.
3. The CAM develops trial and error approachin solving mathematics problems.
4. The method is constructivist is nature as students experience that knowledge created by them may not be permanent.
5. The CAM develops in students the quality ofbeing open-minded to other people's thoughts and ideas.
6. The CAM facilitates higher order thinking.
7. Formulation of hypotheses can be interesting and exciting for the students.
8. The concepts developed with CAM are durable and long-lasting.
9. The students' learning with CAM can be enjoyable.
10. The CAM can be used for all classes and all ages.
11. The CAM helps in analyzing the students' thinking strategy.

Disadvantages of CAM

1. The students who are less likely to take risk may not fully participate in CAM.
2. All students may not be able to analyse positive and negative examples and thereby formulate hypothesis.
3. The teachers may not be able to find enough examples. Test-books are not written on the lines of CAM.
4. The teachers may not be able to analyse students' thinking properly.
5. The students may get disappointed on finding that their hypotheses are not correct.

Although many efforts are made to enhance the mathematics learning of students, mathematics is still considered as a difficult subject by the students. The CAM can help in improving the understanding of students in mathematics concepts.With carefully chosen examples, it is possible to use concept CAM to teach most of mathematics concepts successfully.

Check Your Progress

Notes :

(a) Write your answers in the space given below.

(b) Compare your answers with those given at the end of the unit.

4. Write steps of Project Method.

....

....

....

....

....

5. How many phases a concept attainment model has?

....

....

5.4 LET US SUM UP

We have discussed approaches, methods and a model for teaching mathematics successfully. But teachers rarely bother to apply them in their classes. The reasons may be any – lack of knowledge, lack of motivation and zeal, overcrowded classes, non-academic workload and so on. The necessity is to apply these approaches in letter and spirit so thattheattainment of the objectives of mathematics teaching becomes reality.

5.5 LESSON-END EXERCISE

1. Describe inductive – deductive approach of teaching mathematics along with merits and demerits.
2. Explain the use of inductive – deductive approach of teaching mathematics with the help of examples.
3. Explain analytic - synthetic approach of teaching mathematics with the help of examples.
4. Illustrate analytic - synthetic approach of teaching mathematics along with merits and demerits.
5. Describe the guided discovery approachof teaching mathematics.
6. Discuss project method of teaching mathematics.
7. How will you employ project method forteaching mathematics? Illustrate and justify.
8. Explain concept attainment model of teaching mathematics. Prepare a lesson plan based upon concept attainment model of teaching mathematics on the topic Rational Numbers.
9. Which is best method of teaching mathematics according to your opinion? Give arguments to support your opinion.

5.6 SUGGESTED FURTHER READINGS

Fitzpatrick, M., & Greer, B. (1990).A project in pure mathematics at A-level. *Teaching Mathematics and Its Applications,* 9(4), 150-158.

Gakhar, S. C. (2006). Teaching of mathematics. Panipat: N.M. Publication.

Joyce, B. & Weil, M. (1986).Models of teaching. New Jersey, Prentice Hall Inc.

Lowry, W.C. (1967). Approaches to discovery learning in mathematics. *Mathematics in the Secondary School*, 50(5), 254-260.

Sidhu, K.S. (2000). The teaching of mathematics. New Delhi : Sterling Publishers Private

Kumar, S. (1997). Teaching of mathematics. New Delhi : Anmol Publications Pvt. Ltd.

5.7 ANSWERS TO CHECK YOUR PROGRESS

1. Inductive method starts from particular and ends at general.

2. Synthetic approach is opposite of analytic approach. In synthetic approach of teaching mathematics, parts are joined to get final solution. Although synthetic approach being elegant and precise, it is not logical in nature.

3. The methods of discovery in mathematics are:
   Inductive method,
   Deductive method, and
   Discovery by analogy.

4. Providing / creating the situation
   Proposing and Choosing the Project
   Planning the Project
   Execution of the Project
   Evaluation of the Project
   Recording and Reporting of the Project

5. Three

—×—

LESSON NO. 6 **UNIT-II**

PEDAGOGIC CONTENT KNOWLEDGE FOR THE TRIGONOMETRY AND COORDINATE GEOMETRY

STRUCTURE

6.1 INTRODUCTION

Every year enormous investments are made by the governments for improving mathematics learning of students. These investments, however fail to make any impact on the students' achievement in mathematics. The reasons may be many.

But one major hindrance, which can be detrimental to the students' achievement in mathematics, is teachers' are poor pedagogic content knowledge.The command a teacher has on mathematics concepts and pedagogy of teaching is correlated for students' achievement in mathematics. Mathematics educators have found significant correlation between teacher's competence and students' achievement in mathematics. In the present lesson, we will learn about pedagogic content knowledge for the trigonometry and coordinate geometry. Also,we will revise certain concept of geometry, trigonometric ratios of complementary angles, and trigonometric ratios of height and distance, already studied by you in the lower classes.

6.2 OBJECTIVES

After you study this lesson, you will be able to:

1. Define pedagogic content knowledge.
2. Describe pedagogic content knowledge in mathematics.
3. Explain pedagogic content knowledge for the trigonometry and coordinate geometry.
4. Define primary conceptsof geometry.
5. Describe trigonometric ratios of complementary angles.
6. Explain trigonometric ratios of height and distance.

6.3 PEDAGOGIC CONTENT KNOWLEDGE FOR THE TRIGONOMETRY AND COORDINATE GEOMETRY

The pedagogic content knowledge (PCK) is a term introduced by Lee Shulman in teaching – learning scenario.PCK is an essential and critical element in determining a teacher's success in handling the teaching and learning process and further produces effective teaching (Hill, Schilling, and Ball, 2004).

According to Shulman (1986),PCK is the understanding of how particular topics, principles, strategies, and the like in specific subjectareas are comprehended or typically misconstrued, are learned and likely to be forgotten.

Loughran, Berry, and Mulhall (2012) stated that PCK an academic idea rooted in the belief that teaching requires considerably more than delivering subject content knowledgeto students, and that student learning is considerably more than absorbing information for later accurateregurgitation.

Carpenter, Fennema, Peterson, and Carey (1988) pointed out thatpedagogical content knowledge includes -

1. knowledge of the conceptual and procedural knowledge that students bring to the learning of a topic,
2. the misconceptions about the topic that they may have developed,
3. the stages of understanding that they are likely to pass through in moving from a state of having little understanding of the topic to mastery of it,
4. the techniques for assessing students' understanding and diagnosing their misconceptions,
5. knowledge of instructional strategies that can be used to enable students to connect what they are learning to the knowledge they already possess,
6. and knowledge of instructional strategies to eliminate the misconceptions they may have developed.' (P. 386).

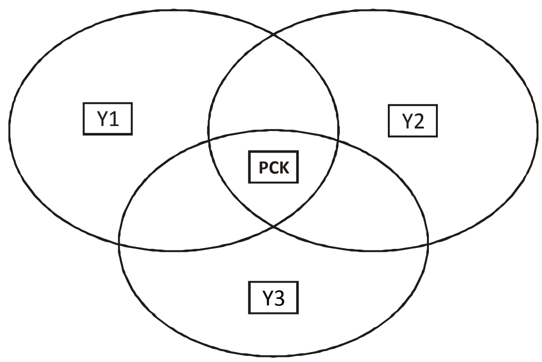
Lucenario, Yangco, Punzalan, and Espinosa (2016) remarked that in most of the previous researches, it has been discovered that pedagogical content knowledge (PCK) is an essential and critical element in determining a teacher's success in handling the teaching and learning processes in the classroom.

Thus, PCK for mathematics is a conceptual framework, which suggests that students'achievement in mathematics depends upon the teacher's content knowldege mastery and teaching expertise. Based upon Shulman (1986) and Baumert and Kunter (2013),PCK for trigonometry and coordinate geometry includes:

PCK for trigonometry and coordinate geometry includes:

1. Knowledge of potential of trigonometry and coordinate geometrytasks and activities to facilitate learning. (Y1)

2. Knowledge of students' beliefs (misconceptions, typical errors, frequently used strategies) and the ability to diagnose students' abilities, prior knowledge, knowledge gaps, and strategies. (Y2)
3. Knowledge acquisition and, in particular, the achievement of a deep understanding of mathematical content are active processes of construction. The teacher can support students' trigonometry and coordinate geometryunderstanding by offering multiple representations and explanations of trigonometry and coordinate geometryconcepts. (Y3)



Thus, PCK trigonometry and coordinate geometry includes teacher's *content knowledge* of trigonometry and coordinate geometry and *pedagogical knowledge* for trigonometry and coordinate geometry. PCK for trigonometry and coordinate geometry is a teacher's perception and understanding of concepts, theories, ideas, knowledge of evidence and proofs of trigonometry and coordinate geometry; deep knowledge of teaching methods,practices, approaches, and strategies; as well as formative and summative assessment techniques. The outline of PCK for trigonometry and coordinate geometry is given here.

PCK for Trigonometry (classes IX and X)

1. ***Concepts, Theories, Ideas, Knowledge of Evidence and Proofsof following***
   (a) Trigonometric Ratios: sine, cosine, tangent of an angle and their reciprocals.
   (b) Trigonometric ratios of standard angles - 0, 30, 45, 60, 90 degrees. Evaluation of an expression involving these ratios.

(c) Simple 2-D problems involving one right-angled triangle.

(d) Concept of sine and cosine being complementary with simple, direct application.

(e) Using Identities to solve/prove simple algebraic trigonometric expressions

- $\sin^2 A + \cos^2 A = 1$
- $1 + \tan^2 A = \sec^2 A$
- $1+\cot^2 A = \text{cosec}^2 A;\ 0 \le A \le 90°$

(f) Trigonometric ratios of complementary angles and direct application:

- $\sin A = \cos(90 - A)$, $\cos A = \sin(90 - A)$
- $\tan A = \cot(90 - A)$, $\cot A = \tan(90 - A)$
- $\sec A = \text{cosec}(90 - A)$, $\text{cosec } A = \sec(90 - A)$

(g) Heights and distances: Solving 2-D problems involving angles of elevation and depression using trigonometric tables.

2. *Teaching methods, approaches, and strategies*

i. Ratio Method and Unit Circle Method (Kendal & Stacey, 1998).

ii. Rusbult's Problem Solving Strategy (Nfon, 2013).

iii. Directed Length Approach and Dynamic Geometry Environments (Hertel & Cullen, 2011).

iv. Engage/Enter, Explore, Explain, Elaborate, and Evaluate (5E) constructivist learning cycle (TUNA &KAÇAR, 2013).

v. Project Method, Laboratory Method,GeoGebra, STAD, and Jigsaw II.

3. Teaching Resources

i. Math Education Community (http://teachingcommons.cdl.edu/mec/index.html) has mentioned some teaching resources for teaching

Trigonometry. These are as follows:

a. Trigonometry for Solving Problems - This lesson offers a pair of puzzles to enforce the skills of identifying equivalent trigonometric expressions. Additional worksheets enhance students' abilities to appreciate and use trigonometry as a tool in problem solving. This lesson is adapted from an article by Mally Moody, which appeared in the March 1992 edition of *Mathematics Teacher*.

b. Trigonometry- This web site has definitions, applets, and much more to help students learn about trigonometry and other connected concepts.

c. Maths Online- Law of Sines- This resource contains multiple applets and graphics that can be used by students or teachers to understand and visualize trigonometry in action.

d. Trigonometry (Applets and Activities)- This resource has multiple applets and activities to be used by the students or teacher for discovery, practice, or review of some basic trigonometric concepts such as definition of sine and cosine, graph of sine and cosine, law of sines, law of cosines, and more.

e. The Math Page: Trigonometry- This resource has multiple concepts for geometry and trigonometry. The concepts are divided among chapters with links on common unknown concepts to help students understand the text. This resource also provides exercises that can be done by the students (answers are provided).

f. Web Math - Students could use this as a resource for review or to clarify topics discussed in class but not fully understood. The resource is easy to use and navigate to either specific concepts or broader topics. The site also offers specific sections on the conversion of units (applicable to the sciences).

ii. Kissane and Kemp (2010) in their article, 'teaching and learning trigonometry with technology' suggested some resources for effective teaching of trigonometry.

iii. GeoGebra Math Apps.

iv. NCERT class IX and class X mathematics text books.

v. Models and charts for teaching trigonometry.

4. Assessment Techniques

Formative assessment – Teacher made as well asstandardized Diagnostic Tests (paper-pencil as well as computer based) for error analysis.

Summative Assessment – Monthly and Annual Test.

PCK for Coordinate Geometry (classes IX and X)

1. Concepts, Theories, Ideas, Knowledge of Evidence and Proofsof following

Cartesian System, Plotting a point in the plane for given coordinates.

(a) Dependent and independent variables.

(b) Ordered pairs, co-ordinates of points and plotting them in the Cartesian Plane.

(c) Graphs of x=0, y=0, x=a, y=a, x=y, y=mx+c including identification and conceptual understanding of slope and y-intercept.

(d) Recognition of graphs based on the above.

2. *Teaching methods, approaches, and strategies*

a. GeoGebra

b. Van Hiele Model of Geometric Thinking

c. Laboratory Method, Project Method, Cooperative Learning

3. Teaching Resources

The Quadrilateral Detective: A Coordinate Geometry Activity - The Allman Files

Triangle Centers Coordinate Geometry : Common Core Performance Task - The Allman Files

Coordinate Geometry: Problem Solving as Applied to Urban Planning – SamizdatMath

Coordinate Geometry Common Core Bundle - Meaningful Math and More

Coordinate Geometry Lesson Plan & Activities – study.com

PDST Post-Primary Maths Team https://www.projectmaths.ie/

The Mathematics Education Community in the CSU - http://teachingcommons.cdl.edu/mec/index.html

NCERT class IX and class X mathematics text books.

Models and charts for teaching trigonometry.

4. ***Assessment Techniques***

Formative assessment – Teacher made as well as standardized Diagnostic Tests (paper-pencil as well as computer based) for error analysis.

Summative Assessment – Monthly and Annual Test.

Check Your Progress

Notes :

(a) Write your answers in the space given below.

(b) Compare your answers with those given at the end of the unit.

1. Who introduced PKC in teaching–learning scenario?

....

2. Define PKC.

....

3. Name one App for helpful for teaching – learning of trigonometry and coordinate geometry.

....

6.4 PRIMARY CONCEPTS

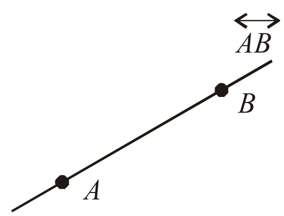
6.4.1 Geometry

The term 'Geometry' has been derived from the Greek word 'Geometron'. 'Geo' means Earth and 'metron' means Measurement. Here, we will learn some basic concepts of geometry.

Point : A point is a location. It has no size i.e. no width, no length and no depth. A point is shown by a dot.

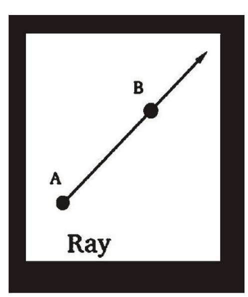
. Point

Line : A line is a straight one-dimensional figure having no thickness and extending infinitely in both directions.A line is uniquely determined by two points, and the line passing through points A and B is denoted $\overleftrightarrow{AB}$.

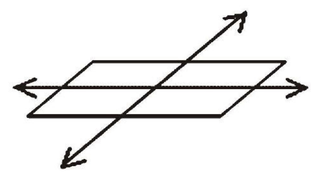


Line Segment : A part of a line that has defined endpoints is called a line segment. A line segment as the segment between A and B above is written as $\overline{AB}$ or $\overline{BA}$.

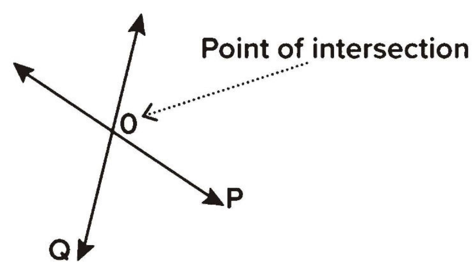
Ray : Ray isa part of a line with a start point but no end point (it goes to infinity). Ray is denoted by $\overrightarrow{AB}$.



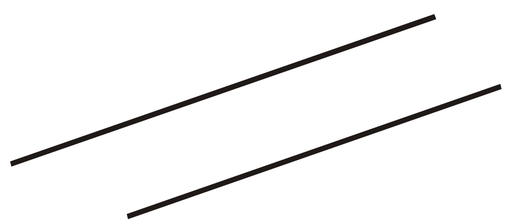
Plane : A plane is a flat surface which is infinitely large and having zero thickness. In order to conceive the idea of plane, think about a flat sheet of metal that is infinitely large in both directions and has zero thickness.The plane is two-dimensional. This is the 'plane' in geometry.



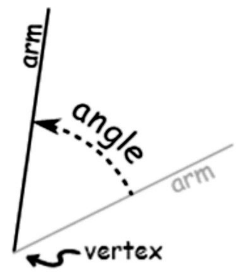
Intersecting Lines : Lines that intersect in a point are called intersecting lines. Two intersecting lines cross each other in a plane. The shared point is called point of intersection.The intersecting lines (two or more) meet only at one point always.



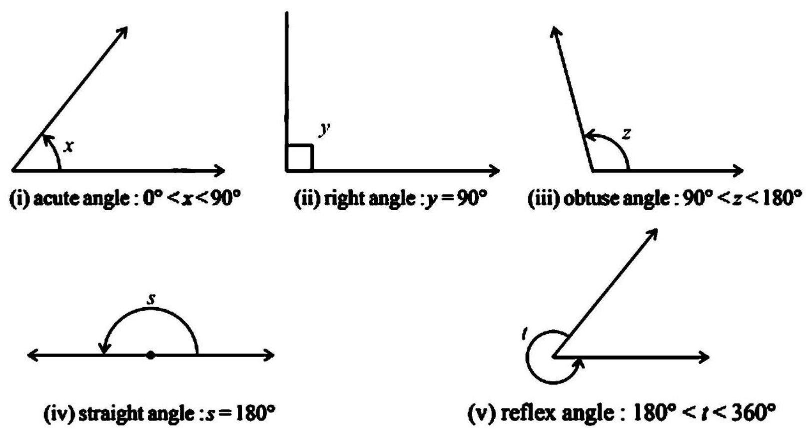
Parallel Lines : Lines on a plane that never meet each other are called parallel lines. The distance between two parallel lines remains same.



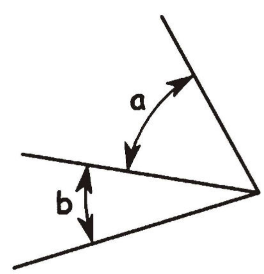
Angle : An angle is formed when two rays originate from the same end point. The point at which the two rays meet (intersect) is called the vertex. The angle measures the amount of turn between the two arms or sides of an angle and is usually measured in degrees or radians.



Types of Angles : Angles can be categorized as acute angle, right angle, obtuse angle, straight angle and reflex angle on the bases of their measure.

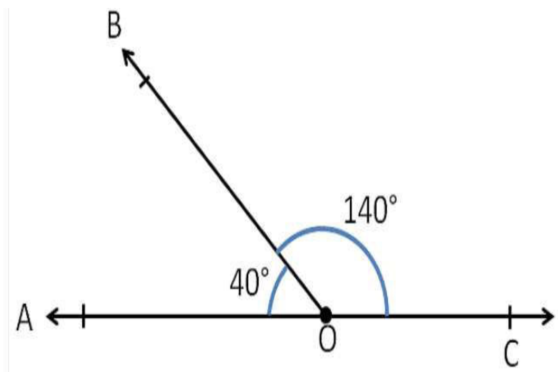


Adjacent Angle : Two anglesare adjacent, if they have a common vertex, acommon arm and their non-common arms areon different sides of the common arm. Here, two adjacent angles are formed when one angle is divided into two parts by a ray.

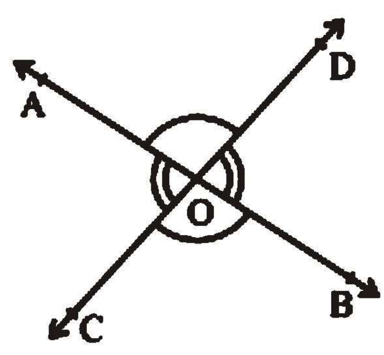


Linear Pair of Angles:

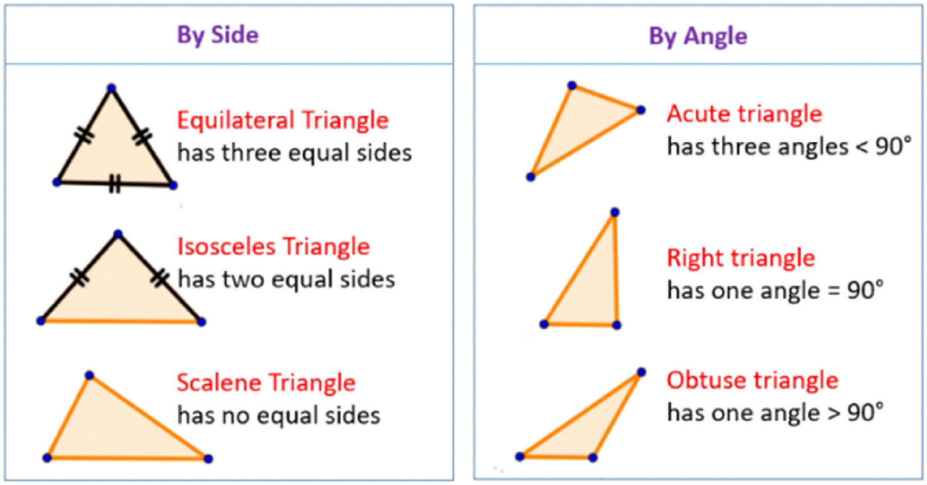
- A linear pair of angles is formed when two lines intersect.
- Two angles are said to be linear if they are adjacent angles formed by two intersecting lines.
- The measure of a straight angle is 180 degrees, so a linear pair of angles must add up to 180 degrees.



Vertically Opposite Angles : Vertically opposite angles formed when two lines intersect each other at a point. There are two pairs of vertically opposite angles in the figure given below $\angle$AOD and $\angle$BOC; and $\angle$AOC and $\angle$BOD respectively.



Triangle : Triangle is a closed figure formed by three intersecting lines. The triangles can be classified on the basis angles and sides.



Congruence of Triangle: Two triangles are called congruent triangles if they are equal in all respects. Congruent triangles are equal with respect to shapes and sizes. The corresponding parts in congruent triangles are equal and we write in short 'CPCT' for corresponding parts of congruent triangles.

Criteria for Congruence of Triangles:

Side-Angle-Side (SAS)–Two triangles are congruent if two sides and the included angle of one triangle are equal to the two sides and the included angle of the other triangle.

Angle-Side-Angle (ASA) – Two triangles are congruent if two angles and the included side of one triangle are equal to two angles and the included side of other triangle.

Angle-Angle-Side (AAS) -Two triangles are congruent if any two pairs of angles and one pair of corresponding sides are equal.

Quadrilateral: A quadrilateral is a plane figure made with four line segments closing in a space.It has four sides, four angles and four vertices. The types of quadrilaterals are given below.

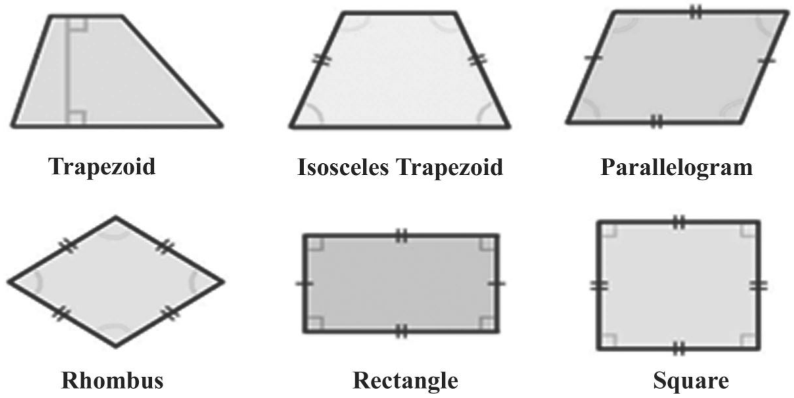
Trapezium is a quadrilateral whose one pair of opposite sidesis parallel.

Parallelogram is a quadrilateral whose opposite sides areparallel.

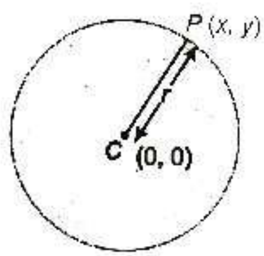
Rectangle is a quadrilateral whose opposite sides are paralleland one angle is a right angle.

Rhombus is a quadrilateral whose opposite sides are paralleland adjacent sides are equal (or a rhombus is a parallelogramwhose all sides are equal in length).

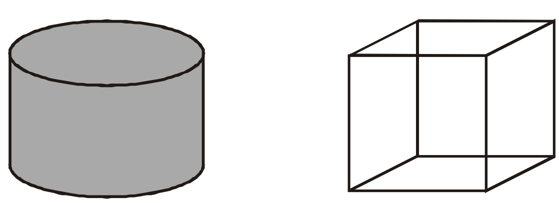
Square is a rectangle whose all sides are equal.



Circle : Circle is defined as the locus of a point which moves in a plane such that its distance from a fixed point in that plane is constant.



Space : Space is the set of all points in the three dimensions - length, width and height. It is made up of an infinite number of planes. Figures in space are called solids.



6.4.2 Trigonometric Ratios of Complementary Angles

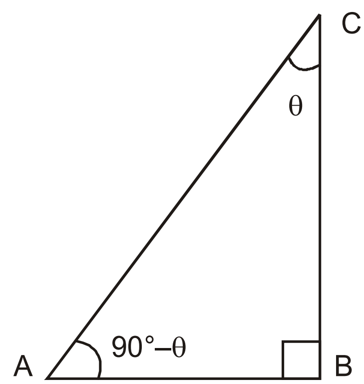
t-ratios of complementary Angles

Complementary Angle : If the sum of two angles is one right angle i.e. 90°, then one angle is said to be complementary of the other angle.

e.g. 40° and 50° , 30° and 60°

So, $\theta°$ and $(90–\theta°)$ are complementary to each other.

Complementary Angle in a right Angled Triangle



Let ABC be a right angled at B

$\therefore$ $\angle ABC = 90°$

Let $\angle ACB = \theta$

We know, by angle sum property of triangle.

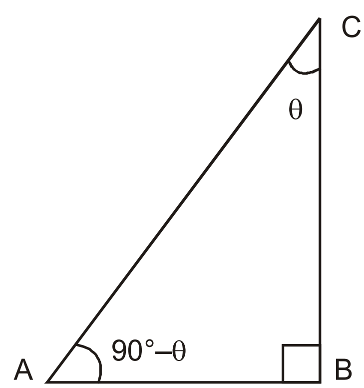
$\angle ABC + \angle ACB + \angle BAC = 180°$

$\Rightarrow$ $90° + \theta + \angle BAC = 180°$

$\Rightarrow$ $\angle BAC = 90° - \theta$

Hence, Angles $\angle BAC$ and $\angle ACB$ are complementary. So, two acute angles of a right angled triangle are always complementary to each other.

Trigonometric Ratios of Complementary Angles



In rt. $\angle$d ΔABC,

For the angle θ, we have

$$\sin\theta = \frac{AB}{AC},\ \cos\theta = \frac{BC}{AC},\ \tan\theta = \frac{AB}{BC}$$

$$\text{cosec}\,\theta = \frac{AC}{AB},\ \sec\theta = \frac{AC}{BC},\ \cot\theta = \frac{BC}{AB}$$

Again, in rt. ∠d. ΔABC, for the angle (90–θ)°, we have

$$\sin(90° - \theta) = \frac{BC}{AC},\ \cos(90° - \theta) = \frac{AB}{AC},\ \tan(90° - \theta) = \frac{BC}{AB}$$

$$\operatorname{cosec}(90° - \theta) = \frac{AC}{BC},\ \sec(90° - \theta) = \frac{AC}{AB},\ \cot(90° - \theta) = \frac{AB}{BC}$$

On comparing both cases :

$$\frac{AB}{AC} = \sin\theta = \cos(90° - \theta) \Rightarrow \sin\theta = \cos(90° - \theta)$$

$$\frac{BC}{AC} = \cos\theta = \sin(90° - \theta) \Rightarrow \cos\theta = \sin(90° - \theta)$$

$$\frac{AB}{BC} = \tan\theta = \cot(90° - \theta) \Rightarrow \tan\theta = \cot(90° - \theta)$$

$$\frac{AC}{BC} = \operatorname{cosec}\theta = \sec(90° - \theta) \Rightarrow \operatorname{cosec}\theta = \sec(90° - \theta)$$

$$\frac{AC}{BC} = \sec\theta = \operatorname{cosec}(90° - \theta) \Rightarrow \sec\theta = \operatorname{cosec}(90° - \theta)$$

$$\frac{BC}{AB} = \cot\theta = \tan(90° - \theta) \Rightarrow \cot\theta = \tan(90° - \theta)$$

Therefore,

sine of any angle = cosine its complementary angle

cosine of any angle = sine its complementary angle

tangent of any angle = Cotangent its complementary angle

cotangent of any angle = tangent its complementary angle

secant of any angle = cosecant of its complementary angle

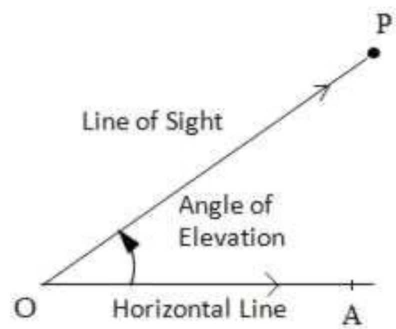
cosecant of any angle = secant of its complementary angle

6.4.2 Trigonometric Ratios of Height and Distance

Trigonometric ratios are used for measuring heights and distances of various objects. Certain primary concepts of trigonometric ratios of height and distance are given here.

Angle of Elevation

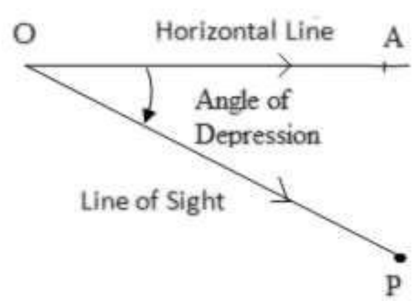
The angle of elevation is the angle between the horizontal and the line from the object to the observer's eye (the line of sight). Suppose you look up at the object P, placed above the level of his eye, from point O then angle of elevation is $\angle AOP$.



Angle of Depression

The angle of depression is the angle between the horizontal and the observer's line of sight. If you are at point O and are looking down at an object P, placed below the level of your eye then angle of depression is $\angle AOP$.

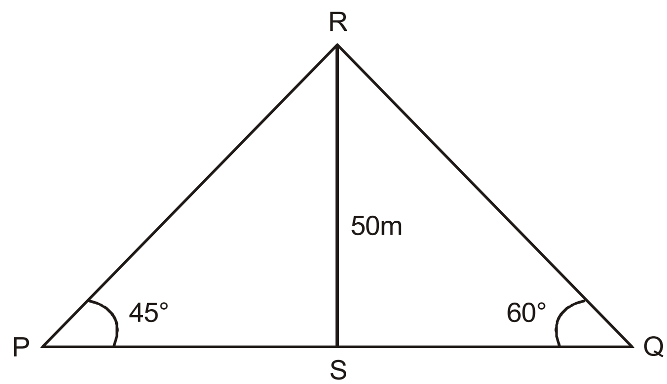
Suppose a man from a point O looks down at an object P, placed below the level of his eye.



Example 1 : Two ships are sailing in the sea on the two sides of a lighthouse. The angle of elevation of the top of the lighthouse is observed from the ships are 45º and 60º respectively. The lighthouse is 50 m high. Find the distance between the two ships.

Solution:

Let SR be the lighthouse and P and Q be the positions of the ships.



Then, SR = 50m, $\angle SPR = 45°$, $\angle SQR = 60°$

Now, $\tan 45° = \frac{SR}{SP}$

$\Rightarrow \quad 1 = \frac{50}{SP}$

$\Rightarrow \quad SP = 50$

Also $\tan 60° = \frac{SR}{SQ}$

$\Rightarrow \quad \sqrt{3} = \frac{50}{SQ}$

$\Rightarrow \quad SQ = 50/\sqrt{3}$

$\Rightarrow \quad SQ = \frac{50}{1.732}$

$\Rightarrow \quad SQ = 28.87$

Therefore, distance between two ships

$$= PQ = SP + SQ = 50 + 28.87 = 78.87$$

Check Your Progress

Notes :

(a) Write your answers in the space given below.

(b) Compare your answers with those given at the end of the unit.

4. What is line?

   ....

5. Define adjacent angles.

   ....

6. A kite is flying at a height of 100 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60°. Find the length of the string, assuming that there is no slack in the string.

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6.5 LET US SUM UP

In this chapter, we studied about pedagogic content knowledge of trigonometry and coordinate geometry and primary concepts in geometry, trigonometric ratios of complementary angles and height and distance. PKC for trigonometry and coordinate geometry is concept that points out that a teacher's expertise in subject matter and pedagogy is necessary for students' learning. PCK for trigonometry and coordinate geometry includes:

1. Knowledge of potential of trigonometry and coordinate geometry tasks and activities to facilitatelearning.
2. Knowledge of students' beliefs (misconceptions, typical errors, frequently used strategies) and the ability to diagnose students' abilities, prior knowledge, knowledge gaps, and strategies.
3. Knowledge acquisition and, in particular, the achievement of a deep understanding of mathematical content are active processes of construction. The teacher can support students' trigonometry and coordinate geometry understanding by offering multiple representations and explanations of trigonometry and coordinate geometry concepts.

6.6 LESSON–END EXERCISE

1. Define Pedagogic Content Knowledge. Illustrate Pedagogic Content Knowledge of trigonometry with the help of concrete examples.
2. Explain Pedagogic Content Knowledge of coordinate geometry.
3. Define Point, Line, Line Segment, Ray, Circle, Quadrilateral, Triangle, and Angle.
4. Explain types of Angles, Triangles and Quadrilaterals.
5. What are trigonometric ratios of complementary angles?
6. A straight highway leads to the foot of a tower. A girl standing at the top of the tower observes a car at an angle of depression of 45°, which is approaching the foot of the tower with a uniform speed. Six seconds later, the angle of depression of the car is found to be 60°. Find the time taken by the car to reach the foot of the tower from this point

7. A house has a window 20 meters above the ground. Across the street from this house, there is a tall pole. The angle of elevation and depression of the top and bottom of this pole from the window are 60^0 and 45^0 respectively. Determine the height of the pole.

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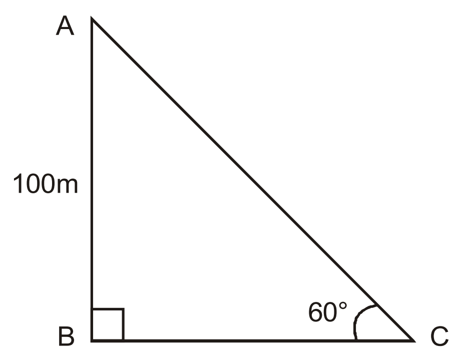
6.8 ANSWERS TO CHECK YOUR PROGRESS

1. Lee Shulman.
2. PCK is the understanding of how particular topics, principles, strategies, and the like in specific subject areas are comprehended or typically misconstrued, are learned and likely to be forgotten.
3. GeoGebra
4. A line is a straight one-dimensional figure having no thickness and extending infinitely in both directions. A line is uniquely determined by two points, and the line passing through points A and B is denoted $\overleftrightarrow{AB}$.
5. Two anglesare adjacent, if they have a common vertex, acommon arm and their non-common arms areon different sides of the common arm. Here, two adjacent angles are formed when one angle is divided into two parts by a ray.
6. Let AC be the length of string



$$\sin\theta = \frac{AB}{AC}$$

$$\Rightarrow \sin 60° = \frac{100}{AC}$$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{100}{AC}$$

$$\Rightarrow AC = \frac{100\times 2}{\sqrt{3}} = \frac{200}{1.73} = 115.60$$

—×—

LESSON NO. 7 **UNIT-III**

FIVE E-MODEL

STRUCTURE

7.1 INTRODUCTION

Techniques of mathematics teaching can be effectively used to make the learning meaningful, interesting and permanent. There is a difference between the method and technique. A method is wider in scope whereas technique is much narrow. Method stands only for a systematic arrangement of the material that is to be taught. But technique is a contributor to the method. Some Important techniques of teaching mathematics are Oral Work, Drill Work, Home Assignments, written Work, Group Work, Self Study, Supervised Study, Review, Brain storming etc. The Present chapter may help in understanding 5 E model, Its five stages, advantages of using 5E model, Drill work, effective use of drill work, advantages and disadvantages of Drill work, Review Work in Mathematics, Reasons for Reviewing, Purpose of Review, Important Points that should be kept in mind while reviewing and suggestions for making effective review.

7.2 OBJECTIVES

After you study this lesson, you will be able to:

1) Describe FiveE Model of Instructions
2) Explain Drill Work in Mathematics teaching
3) Explain Review in Mathematics teaching

7.3 FIVE E MODEL

This is a constructivist model of learning having five stages and each stage begins with the letter E : engage, explore, express, expand and evaluate. Each stage of instruction details the ideas, concepts and skills needed for student inquiry. In addition, there are expected behaviours for teachers and students, as well as opportunities to demonstrate learning through application. It is an alternate way to design instruction to help maximize student engagement in learning. In this model, students construct knowledge and meaning from their experiences. The 5 E method is an example of inquiry-based learning, in which students ask questions, decide what information enhances their understanding, and then self-assess. The strength of the 5E model is that it provides multiple opportunities for assessment and differentiation.

7.3.1 Engage Stage:

The first step in the 5E approach is to engage students in a new skill, idea, or concept. To engage students, teachers should connect the topic or concept at hand with prior understanding. Students are encouraged to ask questions or draw on experiences. The teacher does not correct any misconceptions about the topic or concept but does make notes about revisiting these misconceptions. The purpose of the engage stage is to get students excited and ready to explore the topic or concept.Students are engaged in the lesson by asking questions on demonstrations, observations, making predictions etc. It is critical that the teacher do not explain or teach anything in this stage.

Important points at this stage

1) Grab learner's attention, identifies misconceptions, assesses prior knowledge, promotes thinking, and raises questions.

2) Makes connections to prior knowledge.

3) Disrupts the students' equilibrium

4) Provides a framework for the lesson

5) strives to create order and certainty

6) Promote self-construction and students will be more willing to explore and attempt new material.

7.3.2 Explore Stage:

After the teacher has engaged students in the new concept, he can begin to let them explore the concept or skill in more depth. This could include reading, watching a video, working with more examples, or observing. Once students are interested, they can begin to investigate the problems associated with the topic or concept. Students pose real questions and develop hypotheses. The key concepts in the topic are identified while teachers provide hands-on activities. Students develop the skills that are needed to test their ideas.Students discuss with peers on demonstrations, observations, making predictions etc. The teacher does not provide direct instruction at this time. Instead, the teacher leads students through inquiry-based questions as students work cooperatively in groups. During this stage, time is given to students to refine their hypotheses as they begin to reflect on the results of their investigations.

Important points at this stage

1) Pose questions that allow students to test ideas

2) Allow students to work together and take charge of their own learning

3) Use critical thinking to solve problems

4) At this stage teacher is merely the facilitator of student interaction and uses effective scaffolding techniques to guide student interactions.

5) Encourage students for working in collaborative groups to form and construct new knowledge.

6) Students should be working in their Zone of Proximal Development.

7.3.3 Express Stage

Students develop explanations for what they have already observed. They define the necessary vocabulary and connect their findings to prior knowledge. The teacher should support student discussion and answer student questions. While this stage is a direct instruction phase, the discussions mean that this new information is shared collaboratively. During this stage, students understand this information through a single example. They will need the time, which is provided in the next stage, to internalize their understanding before comparing and contrasting. Students express on the concept based on the teacher's guidance.

Important points at this stage

1) Encourage the Students to share results
2) Discussions should be guided by the teachers
3) Teacher should clarified the results and correct the misconceptions
4) Creates meaning of the lesson and correlates directly with engagement and exploration therefore allowing proper construction of knowledge.
5) Teacher should explain and make connections to the content closely related to lecture base instructions.

7.3.4 Expand Stage

Students need to solidify their understanding by connecting what they have learned to something real. They need to move from a single example in the Express stage to a generalization that can be applied in other examples. In applying this information, students may formulate new hypotheses. The new hypotheses can be tested in new investigations. In practicing new skills, students may take data and make new conclusions. In the investigations during the expand phase, students use the vocabulary and concepts in their discussions and their writing. At this stage Student justifies their views with further explanation and teachers bridges the gap between old and new concept of the student

Important points at this stage

1) Displays deep conceptual understanding
2) Make content more challenging
3) Make students apply knowledge to new contexts which displays deep conceptual understanding.
4) Increase meaningfulness of everything students have learnt
5) Should be based on Constructivist Learning Theory

7.3.5 Evaluate Stage

In the final stage, students return to the engage phase to compare their earlier understanding of what they know now. They address any misconceptions they held, and the teacher makes sure these misconceptions are corrected. They reflect on what they know, and how they are now able to prove what they know in writing, discussion, and demonstration. This evaluate stage should not be skipped. A unit test is not a part of this stage since the teacher can complete a formal evaluation after the evaluation stage. Instead, teachers can evaluate the learning that has taken place through a problem scenario where students should apply their new knowledge. Other evidence of understanding can be done through formative, informal performance, or summative assessments.

Important points at this stage

1) Can be Formal or Informal
2) Check for comprehension
3) Can occur at different points in the lesson
4) The objectives should be properly assessed
5) Evaluation should ensure instructional alignment so that the evaluation will be effective and minimize student confusion.

6) The opportunity for the teacher to form self-reflective practices or the practice of conducting a critical self-examination of one's teaching.

7.3.6 Advantagesof FiveE Model

1) It takes into consideration the different learning styles. i.e. hands on learner, auditory, etc
2) It is a Student-focused learning rather than teacher-focused
3) It easily allows checkpoints for comprehension
4) It sets up for inquiry in the classroom
5) It Increases intrinsic motivation
6) Facilitates collaborative learning
7) Provides opportunities for critical thinking

Check Your Progress

Notes :

(a) Write your answers in the space given below.

(b) Compare your answers with those given at the end of the unit.

1. Which stage is direct instruction stage in 5E model

....

....

2. Give two advantages of 5E model

....

....

7.4 DRILL WORK

Drill is an old method in teaching mathematics especially in arithmetic. The purpose of this method is student able to calculate speedily and accurately. It is an instructional strategy that promotes the acquisition of knowledge or skill through repetitive practice. It refers to small tasks such as the memorization of Tables or the practicing of arithmetic facts. It involves repetition of specific skills, such as addition and subtraction. To be meaningful to learners, the skills built through drill should become the building blocks for more meaningful learning. Drill activities help learner master materials at his own pace. Drills are usually repetitive and are used as a reinforcement tool. Effective use of drill and practice depends on the recognition of the type of skill being developed, and the use of appropriate strategies to develop these competencies. There is a place for drill mainly for the beginning learner or for students who are experiencing learning problems. Its use, however, should be kept to situations where the teacher is certain that it is the most appropriate form of instruction.

According to Lim, Tang and kor (2012): The term drill and practice is defined as a method of instruction characterized by systematic repetition of concepts, examples, and practice problems. Drill and practice is a disciplined and repetitious exercise, used as a mean of teaching and perfecting a skill or procedure. As an instructional strategy, it promotes the acquisition of knowledge or skill through systematic training by multiple repetitions, rehearse, practice, and engages in a rehearsal in order to learn or become proficient. Similar to memorization, drill and practice involves repetition of specific skills, such as spelling or multiplication. To develop or maintain one's specific skills, the sub skills built through drill and practice should become the building blocks for more meaningful learning. Schofield, (1972) defines drill, as "the formation of good or bad habits through regular practice of stereotyped exercises". The habits he talks about seem to be 'at heart' while through practical exercise,' which means the mind adopts the habits and manifest them actively with limbs as instruments.

Drill implies prior understanding of content and its appropriate application. It is one of the important devices of fixing an impression on the minds of students. This is based on the principle of learning by doing and on the law of exercise. Sufficient drill work helps to retain the facts in mind of students.

7.4.1 Effective Use of Drill Work

Although, a certain amount of formal drill is inevitable, preference should be given to functional or meaningful drill. Meaningful drill implies prior understanding of content and its appropriate application. This drill is purposeful and is determined by need as well as by use. An effective drill lesson should be organized keeping in view the following considerations**:**

1) Drill should follow learning and understanding of basics. It should not encourage rote memorization without understanding the subject matter.
2) Drill should be varied. Some routine procedures make learning monotonous and uninteresting.
3) Drill should be individualized and rewarding to each pupil. Each child should see its purpose and utility.
4) Drill periods should be short and the learner's achievement should be frequently tested.
5) Drill should not be planned merely to keep pupils "busy". It should be based upon thought-provoking situations to avoid the repetition of any process mechanically.
6) Drill may also provide diagnostic information about pupils.
7) The material to be drilled upon should be meaningful.
8) The child put on drill exercises, should be fully oriented both in the method and the content of drill performance

9) It should not only develop knowledge and skills but help maintain good habits when they are established

10) It should be properly supervised

11) Drill work should never be given as punishment

12) While evaluating drill work, the accuracy should always be given more importance than the speed of work.

13) While giving drill work, the principle of individual differences must be given due consideration.

14) While giving drill work goals and objectives of teaching mathematics must be kept in mind.

15) Drill work should always be based on the facts already taught.

7.4.2 Advantages of Drill work

1) It is helpful in strengthening the knowledge of the students

2) Students benefit from practice because they are able to apply knowledge through interaction.

3) Students connect with the material when they work with texts and concepts beyond a one-time exposure.

4) When students practice using the knowledge through application, they connect with information on a deeper level.

5) it gives an opportunity to the teacher and the authorities to award proper position to the students in the class

6) It helps the teacher to know the weakness of the students and provide individual attention to them.

7) Students get opportunity to work independently

8) It helps in developing self-confidence of the students

9) Foundation of learned material becomes solid through drill work

10) It is very useful for the average and small students.

7.4.3 Disadvantages of Drill Work:

1) Teachers need to make sure that when having students practice, there is a clear link between concept and action.

2) Drills are not effective when students are not prepared enough; they will not be able to maintain a pace if they are still unclear about a concept.

3) Drills are typically for more basic knowledge. If teaching about more abstract concepts, a drill methodology would not be appropriate.

4) If the exercises are not properly given to the students, it is likely to jeopardized the interest of the students in subject

5) If not properly handled, it is possible that the students may ask someone else to help them in their work. This would make them dependent upon others.

6) It is not very useful for the bright and elder students.

7.5 REVIEW

Review means to view again. Its main purpose is recalling the past experiences to produce better retention. It is a mental process of going over the learnt materials .it is one of the important and effective fixing devices. The previous experiences are recalled for better memory and retention. It involves new relationship and recognition of old materials. It should not be the purpose of repeating the material learned. The review must involve in new learning.

In the words of bossing "the term review connotes not a mere repetition to fix them more firmly in mind, but rather a new view of these facts in different setting that results in understanding, changed attitudes, or different behaivour patterns"

Risk says “review means getting a new view or renewal of an old view to assure a better view or grasp of relationship studied”

Review helps students to assimilate or consolidate what they have learned, enabling them to fit ideas into new patterns. Review assures that the prerequisites needed for learning new content have been mastered. Review adds to students’ confidence in their ability to move successfully to new mathematical topics.

7.5.1 Reasons for Reviewing

Review should be incorporated into mathematics instruction for the following reasons

1) It helps In retention of content.
2) Review promotes continuity and helps students to attain a more comprehensive view of the mathematical topics covered.
3) It helps them summarize main ideas, develop generalizations, and get an overall view of what they have been learning.
4) Review helps students to assimilate or consolidate what they have learned, enabling them to fit ideas into new patterns.
5) Review serves as a diagnostic tool, revealing weaknesses and strengths to students and teachers. It helps teachers identify what is already known and what is not yet known; then re teaching can be planned.
6) Review assures that the prerequisites needed for learning new content have been mastered.
7) Review adds to students’ confidence in their ability to move successfully to new mathematical topics.

7.5.2 Purpose of Review:

1) It focuses on the main points rather than details
2) It develops new interest in old material

3) It introduces new elements and reorganisation of thoughts

4) Its main aim is to fix the concept

5) It reveals student weakness in preparation and understanding

6) It reveals teachers weakness in in planning and teaching

7) It helps the students to organise the materials and experiences into larger units

8) It helps the students to get a broader perspective of what is being studied and the subject matter field as a whole

7.5.3 Important Points that should be kept in mind while Reviewing

1) Review can be done by Outlining. The process of outlining forces students to organize ideas and provides a structure that will help students put ideas together. When students use the outline they have made as an aid in reviewing, restructuring or recall of the mathematical ideas is promoted. Teacher can also use the review questions for reviewing the Mathematical concepts.

2) The teacher should review what has been already covered on the subject at the beginning of the period. This can be done by summarizing the previous lesson and assignment on it relating it to the lesson in hand. This can be done even by asking re capitulatory type of questions

3) Review should be systematically planned and incorporated into the instructional program. Before a new topic or unit is begun, an inventory can help the teacher ascertain whether any prerequisite knowledge is missing. Such a review also helps students to pull together the mathematical ideas they will need for the new topic. The inventory should include the range from simpler skills and concepts to the most difficult in order to pinpoint those which need to be retaught.

4) After completing the full unit, A teacher should review the whole unit, For example after completing the unit on angles, teacher should review it before moving to the unit of triangles as one cannot understand the concept of triangles until one has mastered the unit on angles.

5) When teacher thinks that students have mastered the concept then he can review the students by providing them opportunities of applying the knowledge gained in new situations or putting into practice all that they have learned in classroom.

6) Long-term retention is best served if assignments on a particular skill are spread out in time, rather than concentrated within a short interval. Review immediately after instruction consolidates the ideas from that instruction, while delayed review aids in the relearning of forgotten material

7) Review can involve both teacher and students, it can be in the form of discussion in which the whole class participates, and it is a sort of mass participation of the entire class in an inter-change of ideas under the guidance of teacher.

8) Teacher canuse the pictorial method in reviewing materials, e.g., preparation of charts, graphs and statistical tables, and the introduction of games and contests are the some of the ways of making a review interesting.

9) Short periods of intensive review are better than long periods.

10) Review should be continuous.

11) After a topic or unit is taught, key points or objectives should be reviewed. Students thus become aware of the major highlights of the lesson, so they can focus on the mathematical skills or concepts that will be needed in future lessons.

7.5.4. Suggestion for making effective review

a. It should lead to new learning

b. It should lead to discovery of inter relationship

c. It should lead to continuity, coherence and unity of subject matter

d. It should be directed to the weak points or doubts of the students

e. Review for main points rather than for details

f. Employ review methods which involve visualization

g. Review both as short and at long intervals

Review is an important component of the mathematics instructional program. It can't be neglected — and it can be made interesting as well as profitable.

Check Your Progress

Notes :

(a) Write your answers in the space given below.

(b) Compare your answers with those given at the end of the unit.

3. What is Drill?

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4. What is Review?

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5. Write two advantages of Drill Work?

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7.6 LET US SUM UP

This lesson discusses the techniques of Mathematics Teaching as the understanding of these techniques helps us in planning lessons effectively. Techniques of mathematics teaching are very important as these help the teacher to transact the contents of Mathematics effectively to the learners. The usage of these techniques enables the teacher to make mathematics teaching more imaginative, creative and interesting for learners. The various techniques discussed are 5E Model, Drill and Review. 5E Model is a constructivist model of learning having five stages and each stage begins with the letter E : engage, explore, express, expand and evaluate. Each stage of instruction details the ideas, concepts and skills needed for student inquiry. The term drill is defined as a method of instruction characterized by systematic repetition of concepts, examples, and practice problems. Drill and practice is a disciplined and repetitious exercise, used as a mean of teaching and perfecting a skill or procedure. Review means getting a new view or renewal of an old view to assure a better view or grasp of relationship studied. Review helps students to assimilate or consolidate what they have learned, enabling them to fit ideas into new patterns.

7.7 LESSON END EXERCISE

1. Explain briefly 5E Model of Instruction?
2. Discuss the Important Points that a teacher should kept in mind while reviewing?
3. What is Drill Work? How as a teacher of Mathematics you can make effective use of Drill Work?

7.8 SUGGESTED FURTHER READING

Burner, J.S., (1961); the Process of Education, Harvard University Press, Combridge.

Merrill, P. F., & Salisbury, D. (1984).Research on drill and practice strategies. Journal of Computer Based Instruction, 11(1), 19–21.Google Scholar

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Piaget, Jean, (1952); The Child's Conception of Nunzber, Routtedge&Kegan Paul Ltd., London.

7.9 ANSWERS TO CHECK YOUR PROGRESS

1) Express Stage

2) a) It takes into consideration the different learning styles. i.e. hands on learner, auditory, etc.

   b) It is a Student-focused learning rather than teacher-focused

3) The term drill and practice is defined as a method of instruction characterized by systematic repetition of concepts, examples, and practice problems. Drill and practice is a disciplined and repetitious exercise, used as a mean of teaching and perfecting a skill or procedure.

4) Review means to view again. Its main purpose is recalling the past experiences to produce better retention. It is a mental process of going over the learnt materials. It is one of the important and effective fixing devices.

5) a) It is helpful in strengthening the knowledge of the students.

   b) Students benefit from practice because they are able to apply knowledge through interaction.

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LESSON NO. 8 **UNIT-III**

ASSIGNMENT TECHNIQUES

STRUCTURE

8.1 INTRODUCTION

Teaching Mathematics needs to know multi techniques, methods, strategies, approaches that break the monotony of the teaching and sustains the interests of learners in learning mathematics. The Present chapter may help in understanding the work in mathematics i.e Assignment techniques, problem solving technique, supervised study technique and oral work in mathematics along with their advantages and disadvantages. This will positively develop the mathematical attitude of the students by which the teacher could make the classroom alive. Each Technique has its own uniqueness and validity in Mathematics teaching.

8.2 OBJECTIVES

After you study this lesson, you will be able to:

1) Describe Assignment Technique in Mathematics
2) Understand Problem solving technique in Mathematics
3) Understand Supervised Study Technique in Mathematics
4) Explain oral work in Mathematics

8.3 ASSIGNMENT TECHNIQUE

Assignment is supplement to classroom teaching. It is a work assigned to the student which may be done at the school or home as desired by the teacher. Every mathematics teacher makes use of assignment technique. There may be pre-lesson or post lesson assignment. It is a sort of undertaking or commitment on the part of learner. He undertakes upon himself the responsibility of carrying out the work assigned.

According to Good's Dictionary of Education (1973) assignment means the act of allotting to class or individuals specific mental or physical tasks.

According to Butler: Assignment is really directed study.

In other words, the whole of prescribed course is divided into number of well-connected portions to be covered in a week or so, are called assignments, thus it is a work allotted to the students to do in a fix time say within a week, fifteen days, one month or two months period. Assignment may consist of solving a few mathematical problems, understanding few propositions, solving exercises based on a proposition, making some construction of geometry, preparing illustrations or models for few topics or learning some formulae by heart etc. There are two types of assignment – home assignment and school assignment.

Home Assignment: It includes writing of answer to questions assigned by the teacher. The teacher gives references from different sources concerning the topic. The students go through the text book and other sources referred by the teacher and grasp the idea of assignment given. They write down the answers to the questions, set by teacher and bring it to the school, hand over the notebook to the teacher. The teacher goes through their answers and find out discrepancies , if there is any, He may ask the student to refer more books if he thinks that the student's answer is not up to mark.

School Assignment : It includes answering questions put by the teacher during the school hours and under the observation of teacher.

Assignment is a sort of self-study which supplements class room teaching and makes the learner self-reliant. In Dalton plan, teaching is wholly based on the assignment system. Allassigned work should be corrected and kept as a cumulative record in notebooks rather than on loose sheets of paper.. Planning the assignment represents one of the most important phases of teaching. It is that part of instructional activity which is devoted to (i) organizing a task to be done, and (ii) fitting to the task an appropriate procedure for accomplishment. It assumes that the most effective learning is the product of self-imposed pupil activity.

8.3.1 Guidingprinciples formaking assignment really effective.

1) The assignment should be clear and definite. It should be brief but fairly explanatory to enable each child to understand the task assigned.
2) An assignment should be in relation to the topic under discussion and not in isolation from it.
3) It should anticipate difficulties in the work to be done, and suggest ways to overcome them.
4) It may arise out of activities, needs and interests of the students.
5) It should connect the new lesson to past experiences and correlate the topic with all related subject matter.
6) It should be interesting, motivating and thought provoking.
7) The activities suggested for the assignment should be varied and adapted to the needs and interests of the students.
8) The aim of assignment must be clear and definite in the minds of the students.
9) It should encourage individual choice and creativity among students.
10) It should foster the habits of working together, planning and execution through democratic ways

11) It should encourage discovery and the use of a variety of sources and ways of learning.

12) It should be well-adjusted to the available time frame of the child and inculcates good study habits.

13) The whole prescribed course should be divided into as many assignments as are permissible under the time at the disposal of the students.

14) Assignment should not be vague and lengthy.

15) Assignment should suit age, intelligence, interest and abilities of the students.

16) Different assignments should be given to the different students of the class.

17) There should be advance planning for the material and the literature to be used.

18) The hints, if any, should be given in a comprehensive way and at an appropriate time.

19) Teacher must tell about the reference books, encyclopaedia, good books etc. regarding the assignment

20) It should be flexible and should be changed according to the situation.

21) It should be a cooperative activity in which the teacher and the pupil take an effective part.

22) Weekly assignments are preferable to daily or monthly ones.

23) The work of assignment should be properly evaluated and suggestions must be given at the end.

24) Too many assignments should not be given.

8.3.2 Advantages of Assignment Technique

1) This technique is based on the principle of learning by doing and provides full opportunity to the students to do work according to their convenient.

2) It removes the difficulties / doubts of the students in learning Mathematics.

3) It develops confidence among the students.

4) When completed successfully, It develops sense of achievement in students.

5) It satisfy student's curiosity and acquisitive instincts.

6) It supplements the classroom teaching.

7) It correlates with the previous knowledge and experiences.

8) It helps in increasing the practical skills of the students.

9) It helps in the retention of learned material.

10) It motivates the students for future work.

11) It develops sense of competition among the students.

12) It develops insight among the student.

13) It improves teacher taught relationship.

14) The students may keep their completed assignment for future reference.

15) The teacher can pay individual attention to the pupil.

16) It helps in developing heuristic attitude among students.

17) It is based on the principle of individual differences as each student work at his/her own pace.

18) It helps in tracking the progress of each student. The progress chart made by the teacher about the progress of each student gives an idea about the weaker and brighter students at a glance.

19) It helps in identifying the errors committed by the students.

20) It is based on the principle of assimilation.

8.3.3 Disadvantages of Assignment Technique

1) Students may copy assignment from the other intelligent students
2) In completing the assignment work, slow learners lags behind
3) Sometimes, It becomes time consuming and burdensome process
4) Assignment work needs to spend more time in seeking information and its retrieval
5) The text books written on these lines are not available
6) Most of the schools lacks well equipped mathematics library which is essential requirement for this method
7) It puts extra work on the part of teacher who is already overloaded
8) It becomes very difficult for the teacher to finish heavy curriculum, if he/ she uses this method frequently

8.4 PROBLEM-SOLVING TECHNIQUE

Problem solving technique consists in training the pupils to solve problem. The teacher presents a problem which challenges the intellect of the students. Here problem itself is a crux of problem. It pre-supposes the existence of a problem in the teaching learning situations. A problem is a sort of obstruction or difficulty which has to be overcome to reach the goal. Problem-solving is an individual or a small group activity, most efficient when done cooperatively with free opportunitiesfor discussion. As a consequence, it permits the incorporation of a wide range of feeling and styles of thinking and development. Problem-solving reflects the process of mathematics. It increases a child's ability to think mathematically. The technique of problem-solving is a method of thinking, of analysing, and of learning how to find the answer to a question or problem using known ideas. Learning through problem-solving is a regression from known ideas to unknown ideas, from old ideas to new ideas and from the simple to the complex.

Problem-solving essentially results in an increased ability to think and generate ideas of mathematics. It provides opportunity to the pupils for analysing and solving a problem faced by him on the basis of previous stock of his knowledge enriched with the present means available to him, quite independently by following some systematic and scientific steps and arriving at some basic conclusions or results to be utilised in future for the solution of the similar problems in the identical situations.

8.4.1 Definitions of Problem Solving Techniques

Polya (1945) - Problem solving is finding a way out of a difficulty, a way around an obstacle, attaining an aim that was not immediately attainable.

Yaakam and Simpson- A problem occurs in a situation in which a felt- difficulty to act is realized. It is a difficulty that is clearly present and recognized by the thinker. It may be purely mental difficulty or it may be physical and involves the manipulation of data. The distinguishing thing about a problem however is that it impresses the individual who meets it for finding a solution. He recognises it as a challenge.

Gagne- Problem solving is a set of events in which human beings were rules to achieve some goals.

Ausubel- Problem solving involves concept formation and discovery learning.

Newell and Simon (1972) - Problem solving is a search for a path between the given and goal state of a problem.

Schoenfeld (1992) - Problem solving in mathematics refers to the process wherein students encounter a problem – a question for which they have no immediately apparent resolution, nor algorithm that they can directly apply to get an answer.

Mayer and Wittrock (2006) - Problem solving is a cognitive processing directed at achieving a goal when no solution method is obvious to the problem solver.

Cai and Lester (2010) – Problem solving refers to mathematical tasks that have the potential to provide intellectual challenges for enhancing students' mathematical understanding and development.

Risk, T.M. defines problem solving as "Planned attack upon a difficulty or perplexity for the purposes of finding a satisfactory solution

Thus, Problem Solving method begins with the statement of a problem that challenges the students to find a solution.

8.4.2 Steps in Problem Solving/ Process of Problem Solving

1) **Sensing, Accepting and Defining a Problem:**The student should be able to sense, accept and define the problem which is intriguing or meaningful to students of the relevant age. The problem need not always be real. The only important factor is acceptance of the problem by children as their own.

2) **Analysing the Problem:** The student should carefully analyse the problem as to what is given and what is to find out. Given facts must be identified and expressed. The student must be clear to what exactly is to be done. The process has to be delimited.

3) **Collecting and organizing relevant data:** Student should recall all sort of relevant data which can be helpful in solving the problem. Then the data should be so organised that it can lead to the solution of problem.

4) **Formulating Tentative Hypothesis:** On the basis of organised data student should formulate the tentative solutions of the problem. It is a preparation of list of possible reasons of the occurrence of problem. Formulating hypothesis develops thinking and reasoning power of students. The focus at this stage is on hypothesizing – searching for tentative solutions of the problem.

5) **Testing The Hypothesis:** It includes pursuing the plan of action to a tentative answer. This includes techniques such as trial and error, defining terms and relationships using empirical arguments and control of variables**.** Appropriate methods should be selected to test the validity of the tentative hypothesis as a solution to the problem.

6) **Verifying the Result:** No conclusion should be accepted without being properly verified. Students are asked to determine their results and substantiate the expected solutions. They are required to verify the conclusion by reversing the process of reasoning.

Problem-solving situations may be used by the teacher for three purposes: (a) for helping children develop mathematical ideas, (b) for the application of known mathematical ideas in new situations, (c) for the analysis of the method of problem-solving. The basic techniques which help are the same for all the three categories. These are Approaches and drawing a diagram, restating the problem in one's own words, dramatizing the situation Techniques ofTeaching or preparing a model, replacing the numbers (quantitative aspects) by variables and Mathematics rearranging data, estimating an answer, arguing backwards logically, i.e. from "to prove" to "what is given" and discover the relationships between the known and the unknown.

8.4.3 Merits of Problem Solving Techniques

1) Problem solving provides a real lifeexperiences to the children.
2) This method is psychological and scientific in nature
3) It prepares the students in problem solving and helpsthem to approach the future problems withconfidence.
4) It stimulates thinking, reasoning and imagination of the students.
5) It helps in developing good study habits among students,
6) This method develops power of expression of the students
7) This method develops group feeling while working together in group
8) It develops habit of independent work and initiative in the students
9) It stimulates intellectual curiosity and motivates the students to exert further.
10) This method is based on the principle of learning by doing. Such learning is well retained in the mind.

11) It helps in developing various qualities like patience, cooperation, self-confidence etc. among students.

12) It helps in developing heuristic attitude among students

13) It is based on the principle of individual differences as each student work at his/her own pace

14) This method is helpful in the development of harmonious relationship between teacher and the taught.

15) It helps in increasing the practical skills of the students

16) It helps in breaking the monotony of the classroom as it is a method of experience based learning.

17) It is especially suitable for mathematics, which is a subject of problems.

18) This method is helpful in getting rid of many teaching learning problems like the problem of indiscipline, assigning the home work, etc.

19) It develops analytical, critical and generalisation abilities of the students.

20) It helps to stimulate the reflective and creative thinking of the students

8.4.4 Demerits of Problem Solving Techniques:

1) Not all students are problem solvers.

2) References and resource materials may bedifficult to compile.

3) This method is not suitable for lower classes as they are not sufficiently mature for the purpose.

4) The text books written on these lines are not available.

5) It puts extra work on the part of teacher who is already overloaded.

6) It becomes very difficult for the teacher to finish heavy curriculum, if he/she uses this method frequently.

7) This method is not very economical; it is waste of time and energy.

8) This method demands too much from the teacher, to follow this method talented teachers are required.

9) There is always doubt that students may reach at wrong conclusion.

10) It is not suitable for all topics of mathematics.

In the end, we can say that this is an effective technique in the hands of an able and resourceful teacher of mathematics.

Check Your Progress

Notes :

(a) Write your answers in the space given below.

(b) Compare your answers with those given at the end of the unit.

1. What is Assignment Technique?

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2. Who Said,"Problem solving is a set of events in which human beings were rules to achieve some goals"

a) Risk

b) Schoenfeld

c) Polya

d) Gagne

8.5 SUPERVISED STUDY TECHNIQUE

Supervised study is the study of assignment work by the students in the presence and under the direct supervision of the teacher. This technique is based on the principal of activity and individual differences. It is self-study on the part of students but in the presence and under the direct supervision of the teacher. It may be conducted in the regular periods or in case of residential schools in after school hours. It can be carried out in the reading room of the library where a teacher may be put on duty to watch and guide the students. It may be conducted in mathematics laboratory. Suppose the teacher is teaching some topic through laboratory or heuristic method then he can ask the students to discover the facts or principles through their own independent efforts. Supervised study is a well-known feature of public schools where the resident students are to assemble at one place after the school hours to study under the supervision of teacher. Supervised study creates a formal and studious atmosphere for self-study of the students. The teacher's presence makes the atmosphere more disciplined and congenial for hard work. Under the arrangement of supervised study, every student has to devote the prescribed time compulsorily for self-study. In mathematics, the students can benefit a lot from supervised study. Even if a student is a day scholar, he may be required to study for some time after the school hours for supervised study. This period may be utilised for disposing of the home work. Sometimes the students cannot do their homework at home because they do not get any time for it or they do not get any guidance for the purpose from the parents or from anybody else. During the long vacations also, if the students could be called to school for a few days for supervised study, they may finish the home work allotted to them and thus may be left with the remaining holidays for full enjoyment. If the school provides extra time for supervised study, the students may not have to carry away any burden of home work with them.

8.5.1 Advantagesof Supervised Study Technique:

1) It creates studious atmosphere for the self-study in the classroom
2) Teacher's presence helps in maintaining the discipline in the class
3) It helps in making the environment congenial for hard work
4) It diminishes the burden of home work on the students

5) The students are in the company of their classmates which makes mutual collaboration possible in the class room.

6) Teacher is available on the spot for help and guidance to the needy students.

7) It introduces regularity in work and ensures sustained progress.

8) It helps in removing the mistakes of the students on the spot and removing their difficulties there and then.

9) It is good device for lazy students and shirkers as they compulsorily have to devote the prescribed time for self-study.

10) Teacher can have a continuous assessment of the progress of the students.

11) It helps in utilizing the leisure time of students.

12) It develops the habit of punctuality and systematic work.

13) It helps in strengthening the relationship of teacher and student

14) This technique is very helpful to those students whose home atmosphere is not congenial.

8.5.2 Disadvantages ofSupervised Study Technique:

1) This technique has overburdened the teacher and demands more time and attention from him/her.

2) In this technique over guidance may be interference in the independent thinking and working of students.

3) In this technique students become over dependent on the teacher.

4) Students lose self-initiative with the too much guidance of teacher.

5) Over dependence on teacher harms in the development of problem solving skills of students.

8.5.3. How to make Supervised Study Technique more Effective

1) A congenial and comfortable atmosphere should be ensured in the place meant for supervised study.
2) Teacher must see that students do not waste their time in useless things.
3) The atmosphere should not be very rigid and formal. There should be more of freedom and facility for mutual exchange and collaboration.
4) Students should not be discouraged for the failure of their attempt.
5) The session for supervised study should not be too long. It should not become tiresome and exhausting.
6) The teacher should actively supervise the student's work instead of sitting passively in class.
7) Teacher should not make the students dependent on him instead of this he should build the confidence of the students by encouraging their independent study.
8) Teacher should not un necessarily interfere with the work of students and helps them to work independently.
9) The students should be exempted forthis; otherwise this will kill the very purpose of the supervised study.
10) Teacher must prepare the record of progress of every student in this supervised study.
11) There must be ample space for developing higher order skills like critical thinking, creativity among students.
12) Teacher must give more attention to the weak students.
13) Supervised study is no substitute of self-study, it should supplement the later.

14) The teacher can take the help of brilliant students to share the work of supervision with him.

15) If need be, more than one teacher may be deputed to supervise the work of the students.

8.6 ORAL WORK IN MATHEMATICS

It is the work, which is done orally without the help of written work. There is some kind of direct application of the results and formula learnt by heart. It is a type of mental exercise in which problems are solved orally without the use of pen. Typical mathematical situations which people meet outside the school demand oral computation or involve a visual impression not completely, described in words. A good deal of socio-economic information requires a quick response to a quantitative data. It is, therefore, desirable to develop a high rate of performance consistent with accuracy. Oral work helps each child work at the optimum rate which insures maximum accuracy for him. In any lesson, both oral and written work should mingle freely. Oral work provides a rapid drill designed to habituate a fundamental process, a mode of thinking, or a set of facts. It helps in completing more work in any given period. Generally, material for oral work should not be read from a textbook, Spontaneity in grasping the data, and organization of thought in a limited time, are important aspects to test the pupil's responses. Therefore, in oral work there is either a mental visualization of the process of calculations done in writing or there is some kind of direct application of the results and formulae learnt by heart. In carrying out oral work students may be asked questions or problems in oral form. They have to solve them mentally without the help of any pen or pencil and then give the answer. The value of oral work in mathematics has been widely accepted by great educationists as it may be seen through the following statement of Dr.Salamatullah-"Mental Arithmetic should be regarded as an essential part of education for citizenship and effective living."

8.6.1 Advantages of Oral work

1) Most of the mathematics we use in our practical/ daily life is oral.

2) It is a backbone of overall work in the Mathematics

3) It develops Mental Alertness

4) It develops quick Thinking

5) Oral work is helpful in fixing the learning of new principles and formulae

6) Teacher may easily pay individual attention by distributing oral questions all over the class.

7) It develops the habit of quick learning and thinking and therefore proves very economical device.

8) It stimulates the interest of students in the study.

9) Student's attention may be easily drawn towards the lesson with the help of suitable oral questions.

10) It helps the teacher in judging whether the students are following the lesson or not.

11) It is a good technique as it also develops quick Hearing

12) It is very effective in the initial stages of Lesson

13) It is very effective in the early classes

14) It is an effective means of maintaining discipline in the class

15) It is very helpful in checking previous knowledge of the students

16) It is very helpful in diagnosing the difficulties and doubts of the students

17) It develops the habit of self-study among the students

18) It develops the habit of exactness and precision among students

19) Oral work encourages healthy competition among the students.

20) It helps the students in mental calculations.

21) It is helpful in sharpening the intelligence of the students.

22) It is helpful in removing the shyness of the students as such students can express themselves orally.

23) It is helpful in removing the difficulties of the students in class

24) Probing questions asked by the teachers are helpful in diagnosing the difficulties of the students.

25) It is also very useful at the revision stage of lesson

8.6.2. Disadvantages of Oral Work:

1) Material learned with oral work cannot be retained for long time.

2) All Problems cannot be solved orally.

3) There is no written record of pupil's responses.

4) It is very difficult to plan oral work by the teacher

5) Sometimes students get confused while answering the question orally. The tension is created by face to face contact with the teacher and even in the presence of peers

6) It is not very useful for the higher classes

8.6.3 How to make oral work more effective

1) The teacher should begin his lesson orally as oral work has an appeal the eye and ear of the child. This appeal has greater value than insistence on written work in the very beginning. Oral work gives a good start to every topic.

2) All new processes and methods should be introduced and fixed in child's mind orally.

3) While planning oral work sufficient attention should be given on individual differences.

4) Oral work should be helpful in removing the difficulties of students.

5) In certain cases when repetition is necessary, oral work should take the form of drill work.

6) Oral work should capture the interest of students and should be source of enjoyment to students.

7) More time to oral work should be devoted in lower classes as compared to higher classes.

8) Care must be taken that oral work must be supplemented by written work to get good results.

9) Oral questions should be distributed to the whole class so that no student feels that he/she is neglected.

10) Teacher should always motivate the students and should not discourage them even if they are not able to respond correctly.

Check Your Progress

Notes :

(a) Write your answers in the space given below.

(b) Compare your answers with those given at the end of the unit.

3. What is Supervised Study Techniques?

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4. What is Oral work in Mathematics?

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5. Write two advantages of Oral Work?

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8.7 LET US SUM UP

This lesson discusses the techniques of Mathematics Teaching as the understanding of these techniques helps us in planning lessons effectively. Techniques of mathematics teaching are very important as these help the teacher to transact the contents of Mathematics effectively to the learners. The various techniques discussed areAssignment Techniques, Problem Solving Technique, Supervised Study Technique and Oral Work in Mathematics. Assignment is supplement to classroom teaching. It is a work assigned to the student which may be done at the school or home as desired by the teacher. Every mathematics teacher makes use of assignment technique. Problem solving technique consists in training the pupils to solve problem. Problem solving in mathematics refers to the process wherein students encounter a problem – a question for which they have no immediately apparent resolution, nor algorithm that they can directly apply to get an answer.Supervised study is the study of assignment work by the students in the presence and under the direct supervision of the teacher. This technique is based on the principal of activity and individual differences. Oral Work in Mathematics is a type of mental exercise in which problems are solved orally without the use of pen. Teachers can utilise these techniques in their teaching of Mathematics and get benefit out of it.

8.8 LESSON END EXERCISE

1. Explain briefly assignment techniques in Mathematics?
2. Discuss the steps of Problem Solving Technique in Mathematics?
3. Discuss how you can make supervised study technique more effective?
4. Discuss the advantages of oral work in mathematics ?

8.9 SUGGESTED FURTHER READING

Burner, J.S., (1961); the Process of Education, Harvard University Press, Combridge.

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8.10 ANSWERS TO CHECK YOUR PROGRESS

1) Assignment is supplement to classroom teaching. It is a work assigned to the student which may be done at the school or home as desired by the teacher. Every mathematics teacher makes use of assignment technique.

2) d) Gagne

3) Supervised study is the study of assignment work by the students in the presence and under the direct supervision of the teacher. This technique is based on the principal of activity and individual differences. It is self-study on the part of students but in the presence and under the direct supervision of the teacher.

4) It is the work, which is done orally without the help of written work. There is some kind of direct application of the results and formula learnt by heart. It is a type of mental exercise in which problems are solved orally without the use of pen.

5) a) 1) Most of the mathematics we use in our practical/daily life is oral.

   b) It develops mental Alertness.

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LESSON NO. 9 **UNIT-III**

RATIO AND PROPORTION

STRUCTURE

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9.5 IRRATIONAL NUMBERS

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9.5.2 Application of Assignment Technique for Irrational Numbers

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9.1 INTRODUCTION

Teaching Mathematics needs to know multi techniques, methods, strategies, approaches that break the monotony of the teaching and sustains the interests of learners in learning mathematics. The Present chapter may help in understanding the Application of Assignment techniques, problem solving technique, supervised study technique and oral work in different topics of mathematics like Ratio and Proportion , Arithmetic Mean, Irrational Numbers, laws of real numbers and integers. This will positively develop the mathematical attitude of the students by which the teacher could make the classroom alive. Each Technique has its own uniqueness and validity in Mathematics teaching.

9.2 OBJECTIVES

After you study this lesson, you will be able to:

1) Apply different techniques likeProblem solving technique, Assignment Technique Supervised Study Technique and oral work technique in Ratio and Proportion

2) Apply different techniques likeProblem solving technique, Assignment Technique Supervised Study Technique and oral work technique in Arithmetic mean

3) Apply different techniques likeProblem solving technique, Assignment Technique Supervised Study Technique and oral work technique in Irrational Numbers

4) Apply different techniques likeProblem solving technique, Assignment Technique Supervised Study Technique and oral work technique in Real Numbers

5) Apply different techniques like Problem solving technique, Assignment Technique Supervised Study Technique and oral work technique in Integers

9.3 RATIO AND PROPORTION

Ratio:

A ratio is the comparison or simplified form of two quantities of the same kind. This relation indicates how many times one quantity is equal to the other; or in other words, ratio is a number, which expresses one quantity as a fraction of the other. E.g. Ratio of 3 to 4 is 3 : 4.**. We generally separate the two numbers in the ratio with a colon (:). Suppose we want to write the ratio of 8 and 12.We can write this as 8:12 or as a fraction 8/12, and we say the ratio is *eight to twelve*.**

A ratio can be written in three different ways and all are read as "the ratio of x to y"

x to y

x:y

x/y

Examples:

Jeannine has a bag with 3 videocassettes**, 4 marbles, 7 books, and 1 orange.**

1) **What is the ratio of books to marbles?**

   Expressed as a fraction, with the numerator equal to the first quantity and the denominator equal to the second, the answer would be 7/4. Two other ways of writing the ratio are 7 to 4, and 7:4.

2) **What is the ratio of videocassettes to the total number of items in the bag?**

   There are 3 videocassettes, and 3 + 4 + 7 + 1 = 15 items total.

   The answer can be expressed as 3/15, 3 to 15, or 3:15.

Equivalent Ratios Let us divide a Pizza into 8 equal parts and share it between Ram and Sam in the ratio 2:6. The ratio 2:6 can be written as 2/6;2/6 = 1/3 We know that 2/6 and 1/3 are called equivalent fractions. Similarly we call the ratios 2:6 and 1:3 as equivalent ratios.

From a given ratio x : y, we can get equivalent ratios by multiplying the terms 'x' and ' y 'by the same non-zero number.

For example

1 : 3 = 2 : 6 = 3 : 9

4 : 5 = 12 : 15 = 16 : 20

If a : b is a ratio then:

Duplicate ratio of (a : b) is (a2 : b2).

Sub-duplicate ratio of (a : b) is (a1/2 : b1/2).

Triplicate ratio of (a : b) is (a3 : b3).

Sub-triplicate ratio of (a : b) is (a1/3 : b1/3).

- **PROPORTION**

  A proportion is a name we give to a statement that two ratios are equal. It can be written in two ways:

- two equal fractions, $\mathbf{\frac{a}{b}=\frac{c}{d}}$

- using a colon, **a:b = c:d**

  When two ratios are equal, then the cross product of the ratios are equal.

  That is, for the proportion, **a:b = c:d, a** x **d = b** x **c**

  Proportion is represented by the symbol '=' or '::'

  If the ratio a : b is equal to the ratio c : d, then a, b, c, d are said to be in proportion.

  Using symbols we write as a : b = c : d or a : b :: c : d

- When 4 terms in proportion, then the product of the two extremes (i.e. the first and the fourth value) should be equal to the product of two middle values (i.e. the second and the third value)

- **FOURTH PROPORTIONAL:** If a : b = c : d, then d is called the fourth proportional to a, b, c.

- **THIRD PROPORTIONAL:** a : b = c : d, then c is called the third proportion to a and b.

- **MEAN PROPORTIONAL:** Mean proportional between a and b is √ab.

- CONTINUED PROPORTION a, b, c are in **Continued Proportion** if a : b = b : c. Here b is called the **Mean Proportional** and is equal to the square root of the product of a and c.$b^2 = a \times c \rightarrow b = \sqrt{ac}$
- a/b = b/c = c/d etc., then a, b, c, d are in Geometric Progression.

9.3.1 Application of Problem Solving Technique for Ratio and Proportion:

Problem solving technique consists in training the pupils to solve problem. The teacher presents a problem which challenges the intellect of the students. Here problem itself is a crux of problem. It pre-supposes the existence of a problem in the teaching learning situations. A problem is a sort of obstruction or difficulty which has to be overcome to reach the goal. Problem-solving is an individual or a small group activity, most efficient when done cooperatively with free opportunities for discussion.

Teacher presents the following Problems and asks the students to solve them and students will solve these problems

Problem 1 : Write any 4 equivalent ratios for 4 : 3.

Sol: Given Ratio = 4 : 3. The ratio in fractional form = 4/3, we can get equivalent ratios by "4" and "3" by multiplying with 2, 3, 4, 5 and get the equivalent fractions of 4/3 are 8/6, 12/9, 16/12, 20/15,

∴ The equivalent ratios of 4 : 3 are 8 : 6, 12 : 9, 16 : 12, 20 : 15

Problem 2 : Distribute Rs. 320 in the ratio 1 : 3.

Sol: 1 : 3 means the first quantity is 1 part and the second quantity in 3 parts.

The total number of parts = 1 + 3 = 4. As 4 parts = Rs. 320

∴ 1 part = 320/4 = 80 ? 3 parts = 3 × 80 = Rs. 240

Problem 3 : Prove that 16 : 12 and 4 : 3 are in proportion.

Sol: The product of the means = 12 × 4 = 48. The product of the extremes = 16 × 3 = 48

As Product of Means = Product of Extremes ∴ 16:12, 4:3 are in proportion.

9.3.2 Application of Assignment Technique for Ratio and Proportion

Assignment is supplement to classroom teaching. It is a work assigned to the student which may be done at the school or home as desired by the teacher. Every mathematics teacher makes use of assignment technique. There may be pre-lesson or post lesson assignment. It is a sort of undertaking or commitment on the part of learner. He undertakes upon himself the responsibility of carrying out the work assigned.

Teacher assigns the questions related to ratio and proportion and asks them to practice. This will help the students to identify their weak areas and help them to improve. The students will be asked to solve this Assignment

1) **An office opens at 9 a.m. and closes at 5.30 p.m. with a lunch break of 30 minutes. What is the ratio of lunch break to the total period in the office?**

2) **The shadow of a 3m long stick is 4m long. At the same time of the day, if the shadow of a flagstaff is 24m long, how tall is the flagstaff?**

3) **In a school, the ratio of the number of large classrooms to small classrooms is 3:4. If the number of small rooms is 20, then find the number of large rooms.**

4) **Samira sells newspapers at Janpath crossing daily. On a particular day, she had 312 newspapers out of which 216 are in English and remaining in Hindi. Find the ratio of**

   (a) the number of English newspapers to the number of Hindi newspapers.

   (b) the number of Hindi newspapers to the total number of newspapers.

5) **The students of a school belong to different religious backgrounds. The number of Hindu students is 288, the number of Muslim Students is 252, the number of Sikh students is 144 and the number of Christian students is 72. Find the ratio of the number of Hindu students to the number of Christian students.**

9.3.3 Application of Supervised Study Technique for Ratio and Proportion

Supervised study is the study of assignment work by the students in the presence and under the direct supervision of the teacher. This technique is based on the principal of activity and individual differences. It is self-study on the part of students but in the presence and under the direct supervision of the teacher. Teacher will ask students to solve these questions under his supervision

Q:1 What is the duplicate ratio of 2 : 3?

Sol: Duplicate ratio of 2 : 3 = $2^2 : 3^2$ = 4 : 9.

Q:2 Triplicate ratio of two numbers is 27 : 64. Find their duplicate ratio.

Sol: Triplicate ratio of two numbers is 27 : 64, so numbers should be $27^{1/3} : 64^{1/3}$. So numbers are in the ratio 3 : 4. So duplicate ratio of 3 : 4 = $3^2 : 4^2$ = 9 : 16.

Q:3 The ratio of two numbers is 25 : 36. Find their sub duplicate ratio.

Sol: Sub duplicate ratio of 25 : 36 = $25^{1/2} : 36^{1/2}$ = 5 : 6.

Q:4 Find the missing number in 3 : 4 = 12 : ____

Sol: Let the missing number is "a". We know that,

Product of means = Product of extremes.

Therefore 3 × a = 4 × 12; By dividing both sides by 3, we get the missing term = (4 × 12)/3 = 16

Q:5 Taking 4 and 16 are means, write any two proportions.

Sol: Given 4 and 16 are means. So, __: 4 = 16: __

The product of Means is 4 × 16 = 64. Hence the product of Extremes must also be 64.

64 can be written as 4 × 16 or 2× 32 etc. Two proportions are 2: 4:: 16 : 32 and 16 : 4 :: 16 : 4.

Q:6 Find the fourth proportional of the numbers 12, 48, 16.

Sol: Let fourth proportional is x. Now as per the concept above the product of extremes should be equal to the product of the means → 12/48 = 16/x → x = 64.

Q:7If 2, 5, x, 30 are in proportion, find the third proportional "x".

Sol: Here x is third proportional. According to the concept 2/5 = x/30 → x = 12.

Q:8 Find the mean proportional of the numbers 10 and 1000.

Sol: Mean proportional between a and b is ?ab. Let the mean proportional of 10 and 1000 be x.

So x = √10x1000 = √10000 = 100.

9.3.4 Application of Oral Work Technique for Ratio and Proportion

It is the work, which is done orally without the help of written work. There is some kind of direct application of the results and formula learnt by heart. It is a type of mental exercise in which problems are solved orally without the use of pen. Typical mathematical situations which people meet outside the school demand oral computation or involve a visual impression not completely, described in words. Teacher will ask questions orally from the students which students will answer.

Teacher's Role	**Student's Response**
The ratio of 8 books to 20 books is	**4:5**
Neelam's annual income is Rs. 288000. Her annual savings amount to Rs. 36000. The ratio of her savings to her expenditure is	**1:8**
There are 'b' boys and 'g' girls in a class. The ratio of the number of boys to the total number of students in the class is:	**b/b+g**
3/5=x/20, find x	**12**
3/8=15/40 true or False	**true**
4 : 7 = 20 : 35 true or false	**true**
0.2 : 5 = 2 : ---------	**50**
If b : a = c : d, then a, b, c, d are in proportion.	**false**
Reshma prepared 18kg of Burfi by mixing Khoyawith sugar in the ratio of 7 : 2. How much Khoya did she use?	**14 kg**
Of the 30 persons working in a company, 10 are men and the remaining are women. Find the ratio of the number of men to that of women.	**1:2**

9.4 ARITHMETIC MEAN :

In mathematics, the arithmetic mean , or simply the mean or average when the context is clear, is the sum of a collection of numbers divided by the count of numbers in the collection. The collection is often a set of results of an experiment or an observational study, or frequently a set of results from a survey. The term "arithmetic mean" is preferred in some contexts in mathematics and statistics because it helps distinguish it from other means, such as the geometric mean and the harmonic mean. **Arithmetic mean is the most commonly used and readily understood measure of central tendency in a data set. In statistics, the term average refers to any of the measures of central tendency. The arithmetic mean of a set of observed data is defined as being equal to the sum of the numerical values of each and every observation divided by the total number of observations.**

Symbolically, if we have a data set consisting of the values a1,a2,a3,a4……….an, then the arithmetic mean will be sum of all entities divided by number of entities

A.M = a1+a2+ a3+…………………..+an/ n

For example, take 34, 44, 56 and 78. The sum is 212. The arithmetic mean is 212 divided by four, or 53.

There are two sisters, with different heights. The height of the younger sister is 128 cm and height of the elder sister us 150cm. So what if we want to know the average height of the two sisters? What if we are asked to find out the mean of the heights? As their total height is divided into two equal parts,

128+150= 278/2 = 139 cm

So 139 cm is the average height of the sisters. Here 150 > 139 > 128. Also, the average value also lies in between the minimum value and the maximum value.

Formula for Arithmetic Mean

The arithmetic mean of a set of values is the quantity commonly called "the" mean or the average. Given a set of samples $\{x_i\}$, the arithmetic mean is

$$\bar{x} \equiv \frac{1}{N}\sum_{i=1}^{N} x_i$$

Mean = Sumofallobservations divided by numberofobservations

Example : The runs scored by Sachin in 5 test matches are 140, 153, 148, 150 and 154 respectively. Find the mean.

Solution: . Runs scored by Sachin in 5 test matches: 140, 153, 148, and 154

Means of the runs = totalruns divided by numberofmatches

Mean = 140+153+148+150+154/5 = 745/5 = 149

9.4.1 Application of Problem Solving Technique for Arithmetic Mean

Problem solving technique consists in training the pupils to solve problem. The teacher presents a problem which challenges the intellect of the students. Here problem itself is a crux of problem. It pre-supposes the existence of a problem in the teaching learning situations. A problem is a sort of obstruction or difficulty which has to be overcome to reach the goal. Problem-solving is an individual or a small group activity, most efficient when done cooperatively with free opportunities for discussion.

Teacher presents the following Problems and asks the students to solve them and students will solve these problems

Problem 1 : Find the arithmetic mean of the first 10 natural numbers.

Solution:

The first 7 natural numbers are 1, 2, 3, 4, 5, 6 , 7,8,9 and 10

Let x denote their arithmetic mean.

Then mean = Sum of the first 10 natural numbers/number of natural numbers

x = (1 + 2 + 3 + 4 + 5 + 6 + 7+8+9+10)/10

= 55/10

= 5.5

Hence, their mean is 5.5.

Problem 2 : If the mean of 6, 8, 10, x, 11 is 12, find the value of x.

Solution:

Mean of the given numbers = (6 + 8 + 10 + x + 11)/5 = (35 + x)/5

According to the problem, mean = 12 (given).

Therefore, (35+ x)/5 = 12

⇒ 35 + x = 12× 5

⇒ 35 + x = 60

⇒ 35 – 35 + x = 60 – 35

⇒ x = 25

Hence, x = 25.

Problem 3 :If the mean of five observations x, x + 4, x + 6, x + 8 and x + 12 is 16, find the value of x.

Solution: Mean of the given observations

= x + (x + 4) + (x + 6) + (x + 8) + (x + 12)/5

= (5x + 30)/5

According to the problem, mean = 16 (given).

Therefore, (5x + 30)/5 = 16

⇒ 5x + 30 = 16 × 5

⇒ 5x + 30 = 80

⇒ 5x + 30 - 30 = 80 - 30

⇒ 5x = 50

⇒ x = 50/5

⇒ x = 10

Hence, x = 10.

148 + 153 + 146 + 147 + 154

Problem 4 : The mean of 40 numbers was found to be 38. Later on, it was detected that a number 56 was misread as 36. Find the correct mean of given numbers.

Solution:

Calculated mean of 40 numbers = 38.

Therefore, calculated sum of these numbers = (38 × 40) = 1520.

Correct sum of these numbers

= [1520 - (wrong item) + (correct item)]

= (1520 - 36 + 56)

= 1540.

Therefore, the correct mean = 1540/40 = 38.5.

Problem 5 : The mean of the heights of 6 boys is 152 cm. If the individual heights of five of them are 151 cm, 153 cm, 155 cm, 149 cm and 154 cm, find the height of the sixth boy.

Solution:

Mean height of 6 boys = 152 cm.

Sum of the heights of 6 boys = (152 × 6) = 912 cm

Sum of the heights of 5 boys = (151 + 153 + 155 + 149 + 154) cm = 762 cm.

Height of the sixth boy

= (sum of the heights of 6 boys) - (sum of the heights of 5 boys)

= (912 - 762) cm = 150 cm.

Hence, the height of the sixth boy is 150 cm.

9.4.2 Application of Assignment Technique for Arithmetic Mean

Assignment is supplement to classroom teaching. It is a work assigned to the student which may be done at the school or home as desired by the teacher. Every mathematics teacher makes use of assignment technique. There may be pre-lesson or post lesson assignment. It is a sort of undertaking or commitment on the part of learner. He undertakes upon himself the responsibility of carrying out the work assigned.

Teacher assigns the questions related to Arithmetic Mean and asks them to practice. This will help the students to identify their weak areas and help them to improve. The students will be asked to solve this Assignment

Q:1 If the mean of five observations x, x + 2, x + 3, x + 4 and x + 5 is 16, find the value of x.

Q:2 The mean of 30 numbers was found to be 28. Later on, it was detected that a number 46 was misread as 26. Find the correct mean of given numbers.

Q:3 The mean of the heights of 6 girls is 132 cm. If the individual heights of five of them are 131 cm, 133 cm, 135 cm, 129 cm and 134 cm, find the height of the sixth Girl.

Q:4 If the mean of 90, 80, 100, x, 120 is 150, find the value of x.

Q:5 Find the mean of the first Ten prime numbers.

9.4.3 Application of Supervised Study Technique for Arithmetic Mean

Supervised study is the study of assignment work by the students in the presence and under the direct supervision of the teacher. This technique is based on the principal of activity and individual differences. It is self-study on the part of students but in the presence and under the direct supervision of the teacher. Teacher will ask students to solve these questions under his supervision

Q:1 The heights of five runners are 160 cm, 137 cm, 149 cm, 153 cm and 161 cm respectively. Find the mean height per runner.

Solution:

Mean height = Sum of the heights of the runners/number of runners

= (160 + 137 + 149 + 153 + 161)/5 cm

= 760/5 cm

= 152 cm.

Hence, the mean height is 152 cm.

Q:2. Find the mean of the first five prime numbers.

Solution:

The first five prime numbers are 2, 3, 5, 7 and 11.

Mean = Sum of the first five prime numbers/number of prime numbers

= (2 + 3 + 5 + 7 + 11)/5

= 28/5

= 5.6

Hence, their mean is 5.6

Q:3. Find the mean of the first six multiples of 4.

Solution:

The first six multiples of 4 are 4, 8, 12, 16, 20 and 24.

Mean = Sum of the first six multiples of 4/number of multiples

= (4 + 8 + 12 + 16 + 20 + 24)/6

= 84/6

= 14.

Hence, their mean is 14.

Q:4. Find the arithmetic mean of the first 7 natural numbers.

Solution:

The first 7 natural numbers are 1, 2, 3, 4, 5, 6 and 7.

Let x denote their arithmetic mean.

Then mean = Sum of the first 7 natural numbers/number of natural numbers

$x = (1 + 2 + 3 + 4 + 5 + 6 + 7)/7$

$= 28/7$

$= 4$

Hence, their mean is 4.

Q:5. If the mean of 9, 8, 10, x, 12 is 15, find the value of x.

Solution:

Mean of the given numbers = $(9 + 8 + 10 + x + 12)/5 = (39 + x)/5$

According to the problem, mean = 15 (given).

Therefore, $(39 + x)/5 = 15$

$\Rightarrow \quad 39 + x = 15 \times 5$

$\Rightarrow \quad 39 + x = 75$

$\Rightarrow \quad 39 - 39 + x = 75 - 39$

$\Rightarrow \quad x = 36$

Hence, x = 36.

9.4.4 Application of Oral Work Technique forArithmetic Mean

It is the work, which is done orally without the help of written work. There is some kind of direct application of the results and formula learnt by heart. It is a type of mental exercise in which problems are solved orally without the use of pen. Typical mathematical situations which people meet outside the school demand oral computation or involve a visual impression not completely, described in words. Teacher will ask questions orally from the students which students will answer.

Teacher's Role	Student's Response
Mean of scores obtained by Ram in different papers 80,85,65,60,90 is	**76**
Mean of a set of observations is the value which	**Is a representative of the whole number?**
What is the average of 50, 53, 50, 51, 48, 93, 90, 92, 91, 90	**70.8**
What is the mean of 71, 72, 70, 75, 73, 74, 75, 70, 74, 72	**72.6**
What is the formula of Mean	$\bar{x} \equiv \frac{1}{N}\sum_{i=1}^{N} x_i$
Mean of 7,5,x,8 is 6 then value of x is	**4**
Mean of 69,65,58,x,60 is 63 find x	**63**
Sohan took 7 math tests in one marking period. What is the mean test score? 89, 73, 84, 91, 87, 77, 94	**85**
A mean is commonly referred to as	**Average**
Find the mean swimming time rounded to the nearest tenth : 2.6 min, 7.2 min, 3.5 min, 9.8 min, 2.5 min	**5.1**

Check Your Progress

Notes : (a) Write your answers in the space given below.

(b) Compare your answers with those given at the end of the unit.

1. What do you mean by proportion ?

   ....

   ....

2. What do you mean by Arithmetic Mean?

   ....

   ....

9.5 IRRATIONAL NUMBERS

An **Irrational Number** is a real number that **cannot** be written as a simple fraction. An irrational number is a number that cannot be expressed as a fraction p/q for any integers $\boldsymbol{p}$ and $\boldsymbol{q}$. Irrational numbers have decimal expansions that neither terminate nor become periodic. Every transcendental number is irrational.

Example : π **(Pi)** is a famous irrational number.

π = 3.14159265358979323846264338327950... (and more)

We **cannot** write down a simple fraction that equals Pi.

The popular approximation of 22/7 = 3.1428571428571... is close but **not accurate**.

In mathematics, the irrational numbers are all the real numbers which are not rational numbers, the latter being the numbers constructed from ratios (or fractions) of integers. When the ratio of lengths of two line segments is an irrational number, the line segments are also described as being incommensurable, meaning that they share no "measure" in common, that is, there is no length ("the measure"), no matter how short, that could be used to express the lengths of both of the two given segments as integer multiples of itself.

Among irrational numbers are the ratio π of a circle's circumference to its diameter, Euler's number e, the golden ratio φ, and the square root of two, in fact all square roots of natural numbers, other than of perfect squares, are irrational.

Irrational numbers are numbers that cannot be expressed as the ratio of two whole numbers. This is opposed to rational numbers, like 2, 7, one-fifth and -13/9, which can be, and are, expressed as the ratio of two whole numbers. When expressed as a decimal, irrational numbers go on forever after the decimal point and never repeat.

- Pi, which begins with 3.14, is one of the most common irrational numbers. Pi is determined by calculating the ratio of the circumference of a circle (the distance around the circle) to the diameter of that same circle (the distance across the circle). Pi has been calculated to over a quadrillion decimal places, but no pattern has ever been found; therefore it is an irrational number.

- e, also known as Euler's number, is another common irrational number. The number is named for Leonard Euler, who first introduced e in 1731 in a letter he wrote; however, he had started using the number in 1727 or 1728. e is a universal number. The beginning of this number written out is 2.71828. e is the limit of (1 + 1/n)n as n approaches infinity. This expression is part of the discussion surrounding the subject of compound interest.

- The Square Root of 2, written as √2, is also an irrational number. The first part of this number would be written as 1.41421356237…but the numbers go on into infinity and do not ever repeat, and they do not ever terminate. A square root is the opposite of squaring a number, meaning that the square root of two times the square root of two equals two. This means that 1.41421356237… multiplied by 1.41421356237… equals two, but it is difficult to be exact in showing this because the square root of two does not end, so when you actually do the multiplication, the resulting number will be close to two, but will not actually be two exactly. Because the square root of two never repeats and never ends, it is an irrational number. Many other square roots and cubed roots are irrational numbers; however, not all square roots are.

  The Golden Ratio, written as a symbol, is an irrational number that begins with 1.61803398874989484820...

Irrational numbers are the numbers that cannot be represented as a simple fraction. It is a contradiction of rational numbers but is a type of real numbers. Hence, we can represent it as R\Q, where the backward slash symbol denotes ‘set minus’ or it can also be denoted as R – Q, which means set of real numbers minus set of rational numbers.

The calculations based on these numbers are a bit complicated. For example, $\sqrt{5}$, $\sqrt{11}$, $\sqrt{21}$, etc are irrational. If such numbers are used in arithmetic operations, then first we need to evaluate the values under root. These values could be sometimes recurring also.

The following are the properties of rational numbers:

- The addition of an irrational number and a rational number gives an irrational number. For example, let us assume that x is an irrational number, y is a rational number, and the addition of both the numbers x +y gives a rational number z.
- While Multiplying any irrational number with any nonzero rational number results in an irrational number. Let us assume that if xy=z is rational, then x=z/y is rational, contradicting the assumption that x is irrational. Thus, the product xy must be irrational.
- The least common multiple (LCM) of any two irrational numbers may or may not exist.
- The addition or the multiplication of two irrational numbers may be rational; for example, $\sqrt{2}. \sqrt{2} = 2$. Here, $\sqrt{2}$ is an irrational number. If it is multiplied twice, then the final product obtained is a rational number. (i.e) 2.
- The set of irrational numbers is not closed under multiplication process, unlike the set of rational numbers.

9.5.1 Application of Problem Solving Technique for Irrational numbers

Problem solving technique consists in training the pupils to solve problem. The teacher presents a problem which challenges the intellect of the students. Here problem itself is a crux of problem. It pre-supposes the existence of a problem in the teaching learning situations. A problem is a sort of obstruction or difficulty which has to be overcome to reach the goal. Problem-solving is an individual or a small group activity, most efficient when done cooperatively with free opportunities for discussion.

Teacher presents the following Problems and asks the students to solve them and students will solve these problems

Problem 1 : Find the irrational numbers between 2 and 3.

We know, square root of 4 is 2; $\sqrt{4} = 2$

and square root of 9 is 3; $\sqrt{9} = 3$

Therefore, the number of irrational numbers between 2 and 3 are $\sqrt{5}$, $\sqrt{6}$, $\sqrt{7}$, and $\sqrt{8}$, as these are not perfect squares and cannot be simplified further. Similarly, you can also find the irrational numbers, between any other two perfect square terms.

Problem 2 : Prove that $\sqrt{2}$ is an irrational number

Suppose, $\sqrt{2}$ is a rational number. Then, by the definition of rational numbers, it can be written that,

$\sqrt{2} = p/q$ …….(1)

Where p and q are co-prime integers and $q \neq 0$ (Co-prime numbers are those numbers whose common factor is 1).

Squaring both the sides of equation (1), we have

$2 = p^2/q^2$

$\Rightarrow p^2 = 2q^2$ ………. (2)

From the theorem, if 2 is a prime factor of p^2, then 2 is also a prime factor of p.

So, $p = 2 \times c$, where c is any integer.

Substituting this value of p in equation (3), we have

$(2c)^2 = 2q^2$

$\Rightarrow q^2 = 2c^2$

This implies that 2 is a prime factor of q^2 also. Again from the theorem, it can be said that 2 is also a prime factor of q.

Since according to initial assumption, p and q are co-primes but the result obtained above contradicts this assumption as p and q have 2 as a common prime factor other than 1. This contradiction arose due to the incorrect assumption that $\sqrt{2}$ is rational.

So, root 2 is irrational.

Problem 3 : Which of the following are Rational Numbers or Irrational Numbers?

2, -.45678…, 6.5, $\sqrt{3}$, $\sqrt{2}$

Solution : Rational Numbers –2, 6.5 as these are terminating numbers.

Irrational Numbers – -.45678…, $\sqrt{3}$, $\sqrt{2}$ as these are non-terminating numbers.

9.5.2 Application of Assignment Technique for Irrational Numbers

Assignment is supplement to classroom teaching. It is a work assigned to the student which may be done at the school or home as desired by the teacher. Every mathematics teacher makes use of assignment technique. There may be pre-lesson or post lesson assignment. It is a sort of undertaking or commitment on the part of learner. He undertakes upon himself the responsibility of carrying out the work assigned.

Teacher assigns the questions related to Irrational Numbers and asks them to practice. This will help the students to identify their weak areas and help them to improve. The students will be asked to solve this Assignment

Q:1 Prove that $\sqrt{3}$ is Irrational Number

Q:2 **Which of the following are Rational Numbers or Irrational Numbers?**

7, -.4125678…, 7.5, $\sqrt{5}$, $\sqrt{7}$

Q:3 Compare $\sqrt{29}$ and $\sqrt{19}$.

Q:4 **Check if below numbers are rational or irrational.**

2, 5/11, -5.12, 0.31

Q:5 Define irrational Numbers

9.5.3 Application of Supervised Study Techniquefor Irrational Numbers

Supervised study is the study of assignment work by the students in the presence and under the direct supervision of the teacher. This technique is based on the principal of activity and individual differences. It is self-study on the part of students but in the presence and under the direct supervision of the teacher. Teacher will ask students to solve these questions under his supervision

Question : 1. Compare √11 and √21.

Solution:

Since the given numbers are not the perfect square roots so the numbers are irrational numbers. To compare them let us first compare them into rational numbers. So,

$(\sqrt{11})2 = \sqrt{11} \times \sqrt{11} = 11.$

$(\sqrt{21})2 = \sqrt{21} \times \sqrt{21} = 21.$

Now it is easier to compare 11 and 21.

Since, $21 > 11$.So, $\sqrt{21} > \sqrt{11}$.

Question : 2. Compare √39 and √19.

Solution:

Since the given numbers are not the perfect square roots of any number, so they are irrational numbers. To compare them, we will first compare them into rational numbers and then perform the comparison. So,

$(\sqrt{39})2 = \sqrt{39} \times \sqrt{39} = 39.$

$(\sqrt{19})2 = \sqrt{19} \times \sqrt{19} = 19$

Now it is easier to compare 39 and 19. Since, $39 > 19$.

So,$\sqrt{39} > \sqrt{19}$.

Question : 3. Compare 5 and ?17.

Solution:

Among the numbers given, one of them is rational while other one is irrational. So, to make comparison between them, we will raise both of them to them to the same power such that the irrational one becomes rational. So,

(5)2 = 5 × 5 = 25.

(√17)2 = √17x × √17 = 17.

Since, 25 > 17. So, 5 > √17.

Question : 4. Rationalize 1/4 + √2.

Solution:

Since the given fraction contains irrational denominator, so we need to convert it into a rational denominator so that calculations may become easier and simplified ones. To do so we will multiply both numerator and denominator by the conjugate of the denominator. So,

1/4+√2×(4–√2/ 4–√2)

⇒ 4–√2 / 42–√2 2

⇒ 4–√2 / 16–2

⇒ 4–√2 / 14

So the rationalized fraction is: 4–√2/ 14.

9.5.4 Application of Oral Work Technique forIrrational Numbers

It is the work, which is done orally without the help of written work. There is some kind of direct application of the results and formula learnt by heart. It is a type of mental exercise in which problems are solved orally without the use of pen. Typical mathematical situations which people meet outside the school demand oral computation or involve a visual impression not completely, described in words. Teacher will ask questions orally from the students which students will answer.

Teacher's Role	Student's Response
Give an Example of irrational Number	**Pi**
Which type of number is this 0.16961504…..	**Irrational**
Which type of number is this 0.301056	**Rational**
What type of number is this 0.432666666…..	**Rational**
If the decimal form of a numberstops or repeats, the number is	**Rational**
If the decimal form of a numberdoes not stop and does not repeat, the number is	**Irrational**
Root 5 is an example of	**Irrational Number**
An irrational number is a number that cannot be written as the ratio of	**Two integers**
Approximate value of π is	**π=3.141592654…….**

9.6 REAL NUMBERS

Real numbers are simply the combination of rational and irrational numbers, in the number system. In general, all the arithmetic operations can be performed on these numbers and they can be represented in the number line, also. Real numbers can be defined as the union of both the rational and irrational numbers. They can be both positive and negative and are denoted by the symbol "R". All the natural numbers, decimals, and fractions come under this category.

Properties of Real Numbers:

1. *Closure:* For all real numbers a,b, the sum a + b and the product $a \cdot b$ are real numbers.

2. *Associative laws:* For all real numbers a,b,c,

   $a + (b + c) = (a + b) + c$ and $a \cdot (b \cdot c) = (a \cdot b) \cdot c$.

3. *Commutative laws:* For all real numbers a,b,

   $a + b = b + a$ and $a \cdot b = b \cdot a$.

4. *Distributive laws:* For all real numbers a,b,c,

   $a \cdot (b + c) = a \cdot b + a \cdot c$ and $(a + b) \cdot c = a \cdot c + b \cdot c$.

5. *Identity elements:* There are real numbers 0 and 1 such that for all real numbers a,

   $a + 0 = a$ and $0 + a = a$, and

   $a \cdot 1 = a$ and $1 \cdot a = a$.

6. *Inverse elements:* For each real number a, the equations

   $a + x = 0$ and $x + a = 0$

   have a solution x in the set of real numbers, called the *additive inverse* of a, denoted by -a.

   For each nonzero real number a, the equations

   $a \cdot x = 1$ and $x \cdot a = 1$

   have a solution x in the set of real numbers, called the *multiplicative inverse* of a, denoted by a^{-1}.

Here are some additional properties of real numbers a,b,c, which can be proved from the properties listed above.

- If $a + c = b + c$, then $a = b$.
- If $a \cdot c = b \cdot c$ and c is nonzero, then $a = b$.
- $a \cdot 0 = 0$
- $-(-a) = a$
- $(-a) \cdot (-b) = a \cdot b$

The commutative and associative laws do not hold for subtraction or division:

$a - b$ is not equal to $b - a$

$a \div b$ is not equal to $b \div a$

$a - (b - c)$ is not equal to $(a - b) - c$

$a \div (b \div c)$ is not equal to $(a \div b) \div c$

Laws of Exponents for Real Number:

Product law

According to the product law of exponents when multiplying two numbers that have the same base then we can add the exponents

$$\mathbf{a^m \times a^n = a^{m+n}}$$

where a, m and n all are natural numbers. Here the base should be the same in both the quantities. For example,

- $2^3 \times 2^4 = 2^7$
- $2^{2/3} \times 2^{1/5} = 2^{2/3 + 1/5} = 2^{(10+3)/15}$. We get, $= 2^{12/15}$
- $(-6)^3 \text{ x } (-6)^2 = (-6)^{3+2} = (-6)^5$

Quotient Law

According to the quotient law of exponents, we can divide two numbers with the same base by subtracting the exponents. In order to divide two exponents that have the same base, subtract the power in the denominator from the power in the numerator.

$$\mathbf{a^m \div a^n = a^{m-n}}$$

where a, m and n all are natural numbers. Here the base should be the same in both the quantities. For example,

- $2^5 \div 2^3 = 2^2$
- $p^6 \div p^2 = p^{6-2} = p^4$

Power Law

According to the power law of exponents if a number raise a power to a power, just multiply the exponents

$$\mathbf{(a^m)^n = a^{m \times n}}$$

Here there is one base a and two powers m and n. For example, $(5^3)^2 = 5^{3\times2} = 5^6$

9.6.1 Application of Problem Solving Technique for Real Numbers

Problem solving technique consists in training the pupils to solve problem. The teacher presents a problem which challenges the intellect of the students. Here problem itself is a crux of problem. It pre-supposes the existence of a problem in the teaching learning situations. A problem is a sort of obstruction or difficulty which has to be overcome to reach the goal. Problem-solving is an individual or a small group activity, most efficient when done cooperatively with free opportunities for discussion.

Problem 1 : Solve : 2(3 + 4)

According to the order of operations rules, we should evaluate this expression by first doing the addition inside the parentheses, giving us

$$2(3 + 4) = 2(7) = 14$$

But we can also look at this problem with the distributive law, and of course still get the same answer. The distributive law says that

$$2(3+4) = 2 \times 3 + 2 \times 4 = 6 + 8 = 14$$

Problem 2 : If $12^5 = 3^t \times 4^t$ calculate the value of t.

If two terms have the same power and they are multiplied then the power can be taken as common $= 12^5 = (3 \times 4)^t = 12^5 = 12^t$. If two terms have the same base, then we can equate their powers.

Therefore value of t= 5

Problem 3 : Find the largest number that will divide 398, 436 and 542 leaving remainders 7, 11, and 15 respectively.

Solution:

Algorithm

398 – 7 = 391, 436 – 11 = 425, 542 – 15 = 527

HCF of 391, 425, 527 = 17

9.6.2 Application of Assignment Technique for Real Numbers

Assignment is supplement to classroom teaching. It is a work assigned to the student which may be done at the school or home as desired by the teacher. Every mathematics teacher makes use of assignment technique. There may be pre-lesson or post lesson assignment. It is a sort of undertaking or commitment on the part of learner. He undertakes upon himself the responsibility of carrying out the work assigned.

Teacher assigns the questions related to Real Numbers and asks them to practice. This will help the students to identify their weak areas and help them to improve. The students will be asked to solve this Assignment

Q:1. Which is the smallest composite number?

Q:2. Prove that any positive odd integer is of the form 6x + 1, 6x + 3, or 6x + 5.

Q:3. Evaluate $2 + 3 \times 6 - 5$

Q:4. What is the product of a non-zero rational number and irrational number?

Q:5. Find the value of t $\mathbf{12^7 = 3^t \times 4^t}$

9.6.3 Application of Supervised Study Technique for Real Numbers

Supervised study is the study of assignment work by the students in the presence and under the direct supervision of the teacher. This technique is based on the principal of activity and individual differences. It is self-study on the part of students but in the presence and under the direct supervision of the teacher. Teacher will ask students to solve these questions under his supervision

Question 1.

HCF and LCM of two numbers is 9 and 459 respectively. If one of the numbers is 27, find the other number. (2012)

Solution:

We know,

1st number × 2nd number = HCF × LCM

$\Rightarrow$ $27 \times$ 2nd number $= 9 \times 459$

$\Rightarrow$ 2nd number $= \dfrac{9 \times 459}{27} = 153$

Question 2.

Find HCF and LCM of 13 and 17 by prime factorisation method. (2013)

Solution:

$13 = 1 \times 13$; $17 = 1 \times 17$

HCF = 1 and LCM = $13 \times 17 = 221$

Question 3.

Find LCM of numbers whose prime factorisation are expressible as 3×5^2 and $3^2 \times 7^2$.

Solution:

LCM $(3 \times 5^2, 3^2 \times 7^2) = 3^2 \times 5^2 \times 7^2 = 9 \times 25 \times 49 = 11025$

Question 4.

Find the value of $(7^3)^2$

$(7^3)^2 = 7^{3\times2} = 7^6$

7 * 7*7*7*7*7 =117649

9.6.4 Application of Oral Work Technique for Real Numbers

It is the work, which is done orally without the help of written work. There is some kind of direct application of the results and formula learnt by heart. It is a type of mental exercise in which problems are solved orally without the use of pen. Typical mathematical situations which people meet outside the school demand oral computation or involve a visual impression not completely, described in words. Teacher will ask questions orally from the students which students will answer.

Teacher's Role	Student's Response
If a + c = b + c, then a =	**b**
For all real numbers a,b, the sum a + b and the product a . b are real numbers. Which property is this	**Closure**
am ÷ an =	**a m-n**
2^3 × 24	**27**
-(-a) =	**a**
a . 0 =	**0**
(53)2 =	**53×2 = 56**
a . (b + c) =	**a . b + a . c**
a . 1 = a and 1 . a = a. **Then 1 is called**	**Multiplicative Identity**
The commutative and associative laws do not hold for	**subtraction or division**

9.7 INTEGERS

An integer is a number that can be written without a fractional component. For example, 21, 4, 0, and –2048 are integers, while 9.75, 5+1/2 , and 2 are not. The set of integers consists of zero (0), the positive natural numbers (1, 2, 3, ...), also called whole numbers or counting numbers, and their additive inverses (the negative integers, i.e., –1, –2, –3, ...). The set of integers is often denoted by a bold face Z ("Z") .

A number with no fractional part (no decimals).

Includes:

- the counting numbers {1, 2, 3, ...},
- zero {0},
- and the negative of the counting numbers {-1, -2, -3, ...}

We can write them all down like this: {..., -3, -2, -1, 0, 1, 2, 3, ...}

Examples of integers: -16, -3, 0, 1, 146

Types of Integers

Integers comes in three types:

- Zero
- Postive Numbers(Natural) and
- Negative Numbers(Negatives of Natural Numbers).

Positive numbers

Positive numbers are those numbers which are having a plus sign (+). Most of the time positive numbers are represented simply as a whole number without the plus sign (+). Every positive number is greater than zero as well as negative numbers. On a number line, positive numbers are represented to the right of zero.

Example : 1,2, 200, 5666,99999999, etc.

Negative numbers

In contrast to positive numbers, negative numbers are numbers symbolized with a minus sign (-). Negative numbers are represented to the left of zero on a number line.

Example : …., -99999, -150, -10, -1.

Zero

Zero is neither a positive nor a negative integer. It is a neutral number i.e. zero has no sign (+ or -).

9.7.1 Application of Problem Solving Technique for Integers

Problem solving technique consists in training the pupils to solve problem. The teacher presents a problem which challenges the intellect of the students. Here problem itself is a crux of problem. It pre-supposes the existence of a problem in the teaching learning situations. A problem is a sort of obstruction or difficulty which has to be overcome to reach the goal. Problem-solving is an individual or a small group activity, most efficient when done cooperatively with free opportunities for discussion.

Problem 1 : Solve the following by distributive property.

I. $35 \times (10 + 2) = 35 \times 10 + 35 \times 2$

$= 350 + 70$

$= 420$

II. $(-4) \times [(-2) + 7] = (-4) \times 5 = -20$ And

$= [(-4) \times (-2)] + [(-4) \times 7]$

$= 8 + (-28)$

$= -20$

So, $(-4) \times [(-2) + 7] = [(-4) \times (-2)] + [(-4) \times 7]$

Problem 2 : Mohan deposits Rs.2,000 in his bank account and withdraws Rs.1,642 from it, the next day. If withdrawal of amount from the account is represented by a negative integer, then how will you represent the amount deposited? Find the balance in Mohan's account after the withdrawal.

Solution :

Step 1 :- Understand the problem

For this step first read your problem carefully. So,

- What do you know from the problem?

Here find 'what is the amount Mohan deposited in his bank account and how much he withdraws'.

- What are you trying to find?

Balance in Mohan's bank account after the withdrawal.

Amount deposited = Rs 2000

Amount withdrawn = Rs 1642

Step 2 :- Plan your strategy

Here we have to find the amount he removed from his account. So,

Balance in Mohan's account = Money deposited - Money withdrawn

Step 3 :- Solve the problem

Now that you have all the known and unknown quantities and you also have a strategy to solve your problem, you can now carefully carry out your calculations.

Balance in Mohan's account = 2000 + (-1642) = 2000 - 1642 = 358

Therefore, balance in Mohan's account after withdrawal is Rs 358.

Step 4 :- Revise

Now in this step check your answer.

9.7.2 Application of Assignment Techniquefor Integers

Assignment is supplement to classroom teaching. It is a work assigned to the student which may be done at the school or home as desired by the teacher. Every mathematics teacher makes use of assignment technique. There may be pre-lesson or post lesson assignment. It is a sort of undertaking or commitment on the part of learner. He undertakes upon himself the responsibility of carrying out the work assigned.

Teacher assigns the questions related to Integers and asks them to practice. This will help the students to identify their weak areas and help them to improve. The students will be asked to solve this Assignment

Q:1. What are integers ?

Q:2. Give examples of Positive integers.

Q:3. Give examples of negative integers

Q:4. Solve . 315 × (103 + 72)

Q:5. Fill in the blanks using < or >.

(a) -7 …… -1

(b) 62 ……. -12

(c) -9 …… -3

(d) 51 …… -27

9.7.3 Application of Supervised Study Technique for Integers

Supervised study is the study of assignment work by the students in the presence and under the direct supervision of the teacher. This technique is based on the principal of activity and individual differences. It is self-study on the part of students but in the presence and under the direct supervision of the teacher. Teacher will ask students to solve these questions under his supervision

Question 1.

Fill in the blanks using < or >.

(a) -3 …… -4

(b) 6 ……. -20

(c) -8 …… -2

(d) 5 …… -7

Solution:

(a) $-3 > -4$

(b) $6 > -20$

(c) $-8 < -2$

(d) $5 > -7$

Question 2.

Solve the following:

(i) $(-8) \times (-5) + (-6)$

(ii) [(-6) × (-3)] + (-4)

(iii) (-10) × [(-13) + (-10)]

(iv) (-5) × [(-6) + 5]

Solution:

(i) (-8) × (-5) + (-6)

= (-) × (-) × [8 × 5] + (-6)

= 40 - 6

= 34

Question 3.

Write five pair of integers (m, n) such that m ÷ n = -3. One of such pair is (-6, 2).

Solution:

(i) (-3, 1) = (-3) ÷ 1 = -3

(ii) (9, -3) = 9 ÷ (-3) = -3

(iii) (6, -2) = 6 ÷ (-2) = -3

(iv) (-24, 8) = (-24) ÷ 8 = -3

(v) (18, -6) = 18 ÷ (-6) = -3

Question 4 : Solve the following:

(i) (-15) × 8 + (-15) × 4

(ii) [32 + 2 × 17 + (-6)] ÷ 15

Solution:

(i) $(-15) \times 8 + (-15) \times 4$

$= (-15) \times [8 + 4]$

$= (-15) \times 12$

$= -180$

(ii) $[32 + 2 \times 17 + (-6)] \div 15$

$= [32 + 34 - 6] \div 15$

$= [66 - 6] \div 15$

$= 60 \div 15$

$= 4$

Question 5

The sum of two integers is 116. If one of them is -79, find the other integers.

Solution:

Sum of two integers = 116

One integer = -79

Other integer = Sum of integer - One of integer = 116 - (-79) = 116 + 79 = 195

9.7.4 Application of Oral Work Technique for Integers

It is the work, which is done orally without the help of written work. There is some kind of direct application of the results and formula learnt by heart. It is a type of mental exercise in which problems are solved orally without the use of pen. Typical mathematical situations which people meet outside the school demand oral computation or involve a visual impression not completely, described in words. Teacher will ask questions orally from the students which students will answer.

Teacher's Role	Student's Response
-32+(-6)	–38
-8-9+17	0
+12-12	0
5-(-3)	8
[(-4)+(-6)]+(-2)	–12
a×0 =	0
(-a) ×1 =	–a
a×b)×c = (a) ×(b×c) is which property	associative property of multiplication
(-4) × (2 + 6) =	–32
(-a) × b =	–(a × b)

Check Your Progress

Notes : (a) Write your answers in the space given below.

(b) Compare your answers with those given at the end of the unit.

3. Give an example of Irrational Number

....

....

4. What do you mean by real numbers ?

....

....

5. What do you mean by positive integers ?

....

....

9.8 LET US SUM UP

This lesson discusses the techniques of Mathematics Teaching as the understanding of these techniques helps us in planning lessons effectively. Techniques of mathematics teaching are very important as these help the teacher to transact the contents of Mathematics effectively to the learners. The various techniques discussed are Assignment Techniques, Problem Solving Technique, Supervised Study Technique and Oral Work in Mathematics. Lesson discusses application of these technique in Ratio and Proportion, Arithmetic Mean, Irrational Numbers, Real Numbers and Integers

9.9 LESSON END EXERCISE

1. Explain briefly assignment techniques for the Ratio and Proportion?
2. Discuss the Problem Solving Technique for Irrational Numbers?
3. Discuss supervised study techniquefor real numbers?
4. Discuss the oralwork techniquefor integers ?

9.10 SUGGESTED FURTHER READING

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9.11 ANSWERS TO CHECK YOUR PROGRESS

1) A proportion is a name we give to a statement that two ratios are equal.

2) In mathematics, the arithmetic mean , or simply the mean or average when the context is clear, is the sum of a collection of numbers divided by the count of numbers in the collection

3) Pi, which begins with 3.14, is one of the most common irrational numbers

4) Real numbers can be defined as the union of both the rational and irrational numbers. They can be both positive and negative and are denoted by the symbol "R". All the natural numbers, decimals, and fractions come under this category.

5) Positive numbers are those numbers which are having a plus sign (+). Most of the time positive numbers are represented simply as a whole number without the plus sign (+). Every positive number is greater than zero as well as negative numbers. On a number line, positive numbers are represented to the right of zero.

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LESSON NO. 10 **UNIT-IV**

MATHEMATICS CLUB

STRUCTURE

10.1 INTRODUCTION

10.2 OBJECTIVES

10.3 MATHEMATICS CLUB

10.4 OBJECTIVES OF MATHEMATICS CLUB

10.5 IMPORTANCE OF MATHEMATICS CLUB

10.6 ORGANISATION OF MATHEMATICS CLUB

10.7 ACTIVITIES OF MATHEMATICS CLUB

- 10.7.1 Activities of the club in respect of teaching of circle
- 10.7.2 Activities of the club in respect of Partition of plane of circle by the circle
- 10.7.3 Activities of the club in respect of Theorems on circle and Chords of a circle

10.8 LEARNING TEACHING OF MATHEMATICS BY CO-RELATING IT WITH SCIENCE AND GEOGRAPHY- AREA, SPEED TIME, VOLUME AND SURFACE AREA

10.9 LET US SUM UP

10.10 LESSON–END EXERCISE

10.1 INTRODUCTION

NCF (2005) emphasised that school mathematics should take place in a situation where: Children learn to enjoy mathematics and the Mathematics should be a part of children's life experience which they talk about. This vision can only be fulfilled if mathematics is not confined to only classrooms. There arises the need for such an organization which can channelize student's energies towards desirable goals. There should be a place of Mathematics club in every school of today to widen the knowledge of students. The Present chapter may help in understanding the concept of mathematics Club, its objectives, importance, organisation of the club and activities that can be included in the club.

10.2 OBJECTIVES

After you study this lesson, you will be able to:

1) Describe Mathematics Club
2) Explain the objectives of Mathematics Club
3) Explain the importance of Mathematics Club
4) Describe organisation of Mathematics Club
5) Explain various activities that can be organised in the club

10.3 MATHEMATICS CLUB

Mathematics club is an organization which caters for the inculcation of heuristic attitude and develops genuine interest in mathematics among students. Mathematicalactivities carried out in mathematics club supplement the work of classroom and put the syllabus on practical bias.Mathematics club is a fun and competitive activity where any student can participate in. It is a great way for students to develop intuitive thinking skills and learn new types of math. In mathematics club Students not only learn new material, but also apply their previous knowledge from school to fun and challenging problems. Mathematics club is to create and maintaining interest in mathematics. Certain activities like discussions, lectures, certain mathematical games can be arranged for this purpose. Besides the academic benefits, mathematics club is also a great way to meet math not as a set of rules or guidelines, but as an art. Math is a subject full of creativity and maths club provides opportunitiesto the students for creativity and developing team work skills.

10.4 OBJECTIVES OF MATHEMATICS CLUB

The objectives of mathematics club can be summarized as under.

1) To Encourage a positive attitude towards mathematics
2) To helps in the proper utilization of leisure time.
3) To arouse and maintaining students' interest in mathematics.
4) To stimulate mathematical curiosity.
5) To provide the students with ample opportunities to develop their explorative, creative and inventive faculties.
6) To inculcate the habit of self-study and independent work among the students.
7) To offer an ideal avenue for a free exchange of mathematical ideas and for frank and helpful criticism of these ideas.
8) To provide an informal and a social atmosphere, which the classroom can hardly provide.
9) To offer excellent opportunities for free consideration of matters of special interest to the members.
10) To develop heuristic and problem solving attitude among the students.
11) To provide opportunities for students to translate the theory into practice and to apply their learning in daily life situations.

12) To imbibe social qualities like co-operation, tolerance, adjustment and open-mindedness, as they work in groups.

13) To meet individual needs of the students as every member gets an opportunity to work in his areas of interest.

14) To supplement classroom learning with the informal knowledge acquired through mathematics club activities.

15) To extend learning beyond the limits of the classroom.

16) To provide first hand experiences to the learners as they participate in model making, arranging for exhibition, field work, laboratory work and so on.

10.5 IMPORTANCE OF MATHEMATICS CLUB

1. It is useful in arousing and maintaining interest in mathematics.
2. It inculcate heuristic attitude among the students.
3. It stimulates the active participation of the students.
4. The knowledge gained by students in various functions of such club activities supplements the classroom teaching.
5. It provides ample opportunities of free discussion to the students and they are benefited from one another's view.
6. It helps in satisfying the instincts, urges, interest and needs of gifted students.
7. It gives the students basic training in such programmes.
8. It is helpful in making proper utilization of leisure time.
9. It helps in developing the habit of self-study among the students.
10. It proves helpful in acquainting the students with the latest knowledge and developments in mathematics.
11. It gives them opportunity to translate the theory into practice and to make use of their learning in day to day life.

12. It provides opportunity to discuss the classroom topics in detail and this helps in knowing more and more about the subject.

13. It gives an opportunity to work together then the students learn the lesson of co-operation.

14. It develops the interest of the students in some mathematical hobbies like solving a puzzle, preparing charts, model etc.

15. It provides an opportunity of listening to experts and greatteachers of mathematics from outside the school.

16. It provides an opportunity to inter school and intra school mathematical competitions.

17. It can organize excursion and visits of mathematical value.

18. It is an agency to prepare and display mathematical illustration.

19. It provides inspiration for independent in such work and thus helps in grooming future mathematicians.

20. Through its various programmes, it provides opportunity to the parent and other members of the community to familiar with the school.

21. It provides the opportunity of leadership to some students.

22. It helps in inculcating mathematical creativity among students.

23. It can help in providingeducational and vocational guidance in respect of mathematics.

24. Through mathematics club, the learning of mathematics becomes joyful.

25. It helps the students to develop their inventive faculties.

26. It develops in children a sense of healthy competition for a better cause.

27. It develops the critical thinking and reasoning power of the students.

28. It can cooperate with the other societies in school for various functions.

29. This can look after the mathematics section of the school magazine.

30. It can provide a forum to those interested in mathematics for mathematical activities.

10.6 ORGANIZATION OF MATHEMATICS CLUB

A Mathematics club if properly organized will be a great help in teaching of mathematics. Such a club should be run by the students under the guidance and supervision of their teacher. For proper running of the club the most important thing is the preparation of a draft constitution of the club. Every mathematics club should have its draftconstitution and every member should abide by it. This draft is prepared by the Mathematics teacher in consultation with the head of the institution. The draft constitution should include decisions and directions about the following aspects

1) The name of the club and its aims and objectives. The name of club should be after some renowned mathematicians such as Ramanujan,aryabhatta etc.
2) The conditions and procedure of becoming a member.
3) Club membership fee or other means of finance.
4) Purpose for which expenditure can be incurred and person competent to approve such expenditure.
5) Nature, place and timing of meetings to be held during the session
6) Types of activities to be taken
7) Election procedure of office bearers of the club
8) Rules to be followed by the members and conditions for expelling of members.

For efficient and successful working of Mathematics club the organization of the club should be as under.

1. Head of the institution as its patron. The patron of the club extends all type of facilities and co-operation for the successful execution of the club.

2. A senior mathematics teacher as its incharge. He helps in making the club self-conducting. He guides and guards the members and does not dictate them. It is on him that the club can be kept in its high pitch of activity and interest. All the success of club depends upon his vision.

3. All other teachers of the mathematics subject will be staff advisers. Other subject teachers interested in mathematics will be associate staff advisers

4. Membership of the club is open to all the mathematics students of the school.

5. Associate-membership may be allowed to some other students interested in Mathematics.

6. The club should have an elected executive committee for the academic year. This should include the following duties of members of maths club.

   a) President/ Chairman

   b) Secretary

   c) Assist. Secretary

   d) Treasurer

   e) Librarian

   f) Storekeeper

   g) Publicity Officer

a) President/Chairman:

He being the elected representative of the students should be asked to preside over all the formal functions organized by the club. He has also to preside over the meetings of the executive committee of the club.

b) Secretary:

He is also an elected member of students and is to look after and maintain a proper record of various activities of the club. He should keep a true record of the meeting of the executive committee. He is also responsible to carry out all correspondence on behalf of the club and to extend invitation to speakers and guests for various functions of the club.

c) Assistant Secretary:

His main role is to assist the secretary in performance of duties, i.e., in the absence of secretary he has to carry out all the functions of the secretary.

d) Treasurer:

He is the person who is responsible for collection of membership fees for the club. He has also to maintain proper account of receipts and expenditure of the club.

e) Librarian:

He looks after the library of the club and carries out his own usual duties like maintain the record of books available with the club.

f) Storekeeper:

He maintains the record of the equipment of the club.

g) Publicity Officer

The publicity officer publicizes the activities of the club through different means like publishing its news in the leading newspapers, display its highlights on the display board etc.

These members of executive committee are expected to extend their active co-operation and participate actively in clubs programme. The membership of the mathematics club should not be imposed upon students and should also not be restricted. It should be voluntary and open to all the students in the school. A nominal fee may be charged from each member and the club should try to tap other sources also. Mathematics teacher should utilize his knowledge and influence to make the programmes of the club success.

Check Your Progress

Notes :

(a) Write your answers in the space given below.

(b) Compare your answers with those given at the end of the unit.

1. Give two objectives of Mathematics Club

   ....

   ....

2. The name of Mathematics club should be after some renowned mathematicians True/False

   ....

   ....

10.7 ACTIVITIES OF MATHEMATICS CLUB:

The activities we select are crucial to the success of our mathematics club and need to be researched and matched to our pupils accordingly. They have to be vibrant, inventive, and energetic and fun which means hands-on, hearts-on and minds-on. Some of the activities that may be carried out are

1) Organising Mathematical games, hobbies, projects, experiments, data collection, discussion, debates and innovations

2) Organizing inter-class, inter-school competitions on some interesting mathematical topics.

3) Arranging a lecture by some renowned mathematics teacher and scholar from outside the school.

4) Celebrating days and events pertaining to the history of mathematics or great mathematicians.

5) Organising discussions about the practical applications of mathematics.

6) Organising recreational activities in mathematics such as puzzles, riddles, catch-problems, number games etc.

7) Making or collecting charts, models, picture, graphs etc. for the mathematics laboratory.

8) Conducting Mathematics related project activities.

9) Preparing items related to mathematics for well magazine.

10) Organizing mathematical exhibitions (or) fairs.

11) Organizing certain outings of mathematical interest such as visits to post-offices, banks, market places, big business concerns etc.

12) Making arrangements to listen to certain radio broadcasts on mathematical topics.

13) Organizing seminars and career course relating to mathematics.

14) Organising lectures by experts of allied subjects and professions.

15) Organising excursions and visits of mathematical value.

16) Decorating and equipping mathematics room and laboratory.

17) Organising general and extra reading in mathematics.

18) Relating the teaching of mathematics with other subjects, life activities and environment.

19) Collaborating with the other clubs and societies in the school functions.

20) Rendering all the possible services to the community.

21) Providing opportunities to the gifted students to show their talent

22) Providing guidance to those students who are backward in mathematics.

23) Improvising and preparing handmade mathematics apparatus.

24) Making arrangements for taking advantages of television talks and lessons concerning mathematics.

25) Making arrangements for open-ended activities that engage children in talk-rich learning conversations about maths.

26) Preparing students for various mathematics competitions like Mathematics Olympiad.

10.7.1 Activities of the club in respect of teaching of circle

Teacher can plan many activities in respect of teaching of circle. Some of them can be

1) **Exploring Different Circular Objects:**Students can explore many circular objects and their parts like centre, radius, diameter, chord etc.

2) **Ratio of Circumference and Diameter:** Students can discover that the ratio of circumference to diameter is constant and is equal to ð. In this activity, students use a numeric approach to see the relationship between circumference and diameter. That is, students compute the ratio of circumference to diameter and then take the average for several objects. Students should measure the circumference and the diameter of the objects that they brought to school. An effective method for measuring the circumference is to wrap a string around the object and then measure the string, or to use flexible measuring tapes. To ensure accuracy, care should be taken to keep the string taut when measuring the circular object. Students should be allowed to select which unit of measurement to use. However students should use the same unit for the circumference and the diameter.Students should record the following information in the Circumference Activity Sheet.

   a) Description of each object

   b) Circumference of each object

   c) Diameter of each object

   d) Circumference divided by diameter.

   After the measurements have been recorded, a calculator can be used to divide the circumference by the diameter.When all groups have completed the measurements and calculations, conduct a whole-class discussion.The value is roughly 3.14, or ð. students can also asked to research the history of ð and its calculation, approximation, and uses.

3) **Preparation of Model / charts:**Students can prepare a model/ chart of circle and its different parts. Students can explore different parts of circle like radius, diameter, chord, sectors , arc, semicircle , segment etc.

4) **Relationship between radius and Diameter:** Students are asked to draw different circle and measure its radius and diameter. Then observe the relationship between the two. Discuss the relationship between a circle's diameter and its radius in the presence of all the students. The diameter of a circle is 2 times the radius or the radius is half of the diameter.

5) **Area of Circle:** Ask the students to cut a circular and a rectangular piece of wood. Make a border on all the four sides of the rectangular piece of wood. Paste a coloured paper on the circular piece. Cut the circle into 16 sectors of equal measurements. Make a trace copy of these sectors and paste them on the rectangular sheet. These sectors so arranged form a rectangle whose dimensions are ð r and r.

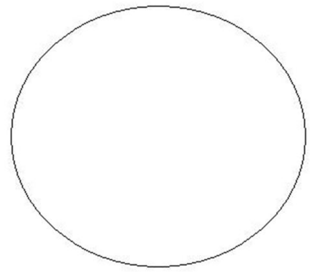
The formula for the area of circle can be explained as below:

Area of rectangle	=	l x b
Here l is π r and b	=	r
Therefore area	=	(π × radius) × (radius)
	=	π × radius2

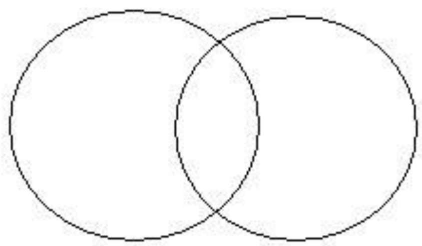
10.7. 2 Activities of the club in respect of Partition of plane of circle by the circle

Finding the equation for the maximum number of regions into which N circles can partition a plane. Students are required to draw and observe

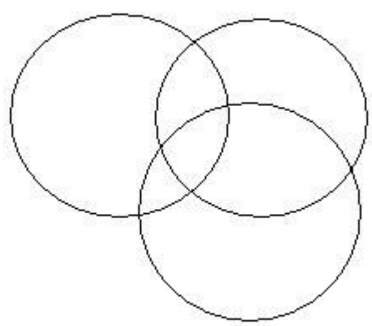
A single circle partitions the plane into two regions, inside and out.



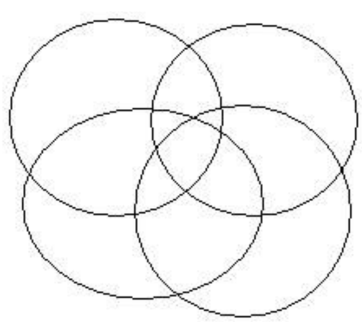
2 circles gives 4 regions



3 Circles gives 8 regions



4 circles gives 14 regions



Students will observe that

When circle is 1 there are 2 regions

When circles are 2 there are 4 regions

When circles are 3 there are 8 regions

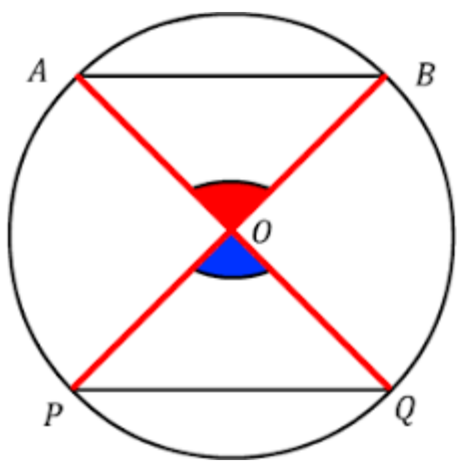
When circles are 4 there are 14 regions

Therefore students can themselves form the equation as

Cn=n2–n+2. (n square –n+2) where n is a number of circles.

10.7.3 Activities of the club in respect of Theorems on circle and Chords of a circle

Theorem: Equal chords of a circle subtend equal angles at the center.



Students are required to prepare a model/ Geo-Board, in which AB and PQ are the equal codes. They have to verify that the angles made by them at the center are equal. Students measure the angles and found them equal. Students are again asked to verify this relationship by drawing different circles. In each case students will find that angle AOB is equal to angle POQ i.e equal chords of a circle subtends equal angles at center.

Theorem : This is the converse of the previous theorem. It implies that if two chords subtend equal angles at the center, they are equal.

Students will again use Geo-board to verify this. Here angle AOB is equal to angle POQ. Now students are required to measure the two chords and will found them equal. Students are again asked to verify this relationship by drawing different circles. In each case students will find that chord AB is equal to chord PQ i.e if two chords subtend equal angles at the center, they are equal.

Similarly with the help of Geo-Board we can easily prove the other theorems likeA perpendicular dropped from the center of the circle to a chord bisects it. It means that both the halves of the chords are equal in length; the line that is drawn through the center of the circle to the midpoint of the chords is perpendicular to it. In other words, any line from the center that bisects a chord is perpendicular to the chord ,Equal chords of a circle are equidistant from the center of a circle. chords equidistant from the center of a circle are equal in length. The angle subtended by an arc at the center of a circle is double that of the angle that the arc subtends at any other given point on the circle etc.

10.8 LEARNING TEACHING OF MATHEMATICS BY CO-RELATING IT WITH SCIENCE AND GEOGRAPHY-AREA, SPEED TIME, VOLUME AND SURFACE AREA

a) Learning Teaching of Mathematics by co- relating it with science:

Mathematics and science have a long and close relationship that is of crucial and growing importance for both. Mathematics is an intrinsic component of science, part of its fabric, its universal language and indispensable source of intellectual tools. Reciprocally, science inspires and stimulates mathematics, posing new questions, engendering new ways of thinking, and ultimately conditioning the value system of mathematics. Different experts have given relationship of mathematics with science in different fashion like

Mathematics is gateway and key to all sciences. Neglect of mathematics work injury to all knowledgesince he who is ignorant of it, cannot know the other science or things of the world and what is worse, men who are thus ignorant are unable to perceive their own ignorance and so do not seek a remedy - Bacon

Mathematics is the indispensible instrument of all physical research- Berthelot

Mathematics is the language of physical sciences and certainly no more marvellous language was ever created by the mind of man-Lindsay

All scientific education which does not commence with mathematics is of necessity, defective at its foundation-Comte

A natural science is a science only in so far as it is mathematical-Kant

Mathematics gives a workable symbolism for brief and precise expression of ideas to all sciences. All laws and principles in science are expressed as equations and formulae usingmathematical language and symbol. A teacher of mathematics while teaching various topics of mathematics like equations and variations makes use of examples from science such an approach make mathematical learning more significant. The process and content of science can inspire an approach to solving problems that applies to the study of mathematics. Mathematics integrates science by using science applications to explain or practice mathematics concepts or to employ the science to reinforce students' interest in mathematics or to enable the students to recognize the broad utility of mathematics. Science depends upon mathematics for the formulation and solution of many of its problems,mathematics also must acknowledge that it acquires meanings as it used to describe the physical world.Science is a body of knowledge about the Universe. Mathematics is a language that can describe relationships and change in relationships in a rational way. Science generally uses mathematics as a tool to describe science. A few scientists like Galileo and Albert Einstein believe that the laws of the Universe are mathematical.

b) Learning Teaching of Mathematics by co- relating it with Geography

Mathematics finds many applications in geography. Geography is nothing but a scientific and mathematical description of our earth in its universe. Plane Euclidean geometry is used in surveying small areas in the field, while spherical geometry and trigonometry are required in the construction of map projections, both traditional elements of mathematical geography. In the newer applications of mathematics to geography, topology is being used increasingly in the spatial analysis of networks. Graph theory provides indices to describe various types of network, such as drainage patterns. Differential equations are needed to study dynamic processes in geomorphology. Statistical techniques, such as trend surface analysis, factor analysis, cluster analysis and multiple discriminant analysis, can be applied to the description and analysis of the data of regional geography. Mathematical models are used in various forms to simplify the problems in geography. Geography makes extensive use of mathematics. Mathematics helps in drawing the Maps to the scales and locating places and estimating the distance between different places. The dimensions and magnitude of earth, its situation and position in the universe, formation of Nights & Days, Solar & Lunar Eclipse, Longitude Latitude, Maximum and Minimum Temperature, Barometric Pressure, Height above Sea Level, Surveying, are some of numerous learning areas of geography which need the application of mathematics. Geographical figures are explained in the terms of numbers only like seasonal conditions, temperature, humidity, degree, measurement of rain etc. The learner needs the help of mathematics in understanding and calculating the local, standard and international time. The surveying instruments in geography have to be mathematically accurate. The geographical conditions also define the economy of a rich/poor country. Many countries like India have agricultural based economy due to its climate, rainfall, rivers and weather prediction.etc.There are changes in the fertility of soil, changes in the distribution of forests, changes in the distribution of population, changes in ecology, etc. which have to be mathematically determined in order to exercise desired control over them. Similarly Geography finds applications in Mathematics as geography deals with natural resources and mathematics determines the balance between their availability and consumption. Geography enables the application of numerical and statistical methods to real-world contexts and issues. Geography helps to develop skills of analysis, synthesis and presentation, drawing on their own field data and secondary sources which are in turn very helpful in mathematics. Mathematics deals with numbers and measurements but number and measurements are based on environment which depends upon geography

c) Area:

Area is a mathematical term defined as the two-dimensional space taken up by an object. It is the space occupied by a flat shape or the surface of an object. The area of a figure is the number of unit squares that cover the surface of a closed figure. Area is measured in square units such as square meters, square feet, square inches, etc. Area is a measure of how much space there is on a flat surface. For example two sheets of paper have twice the area of a single sheet, because there is twice as much space to write on.The origin of the area is from Latinword, meaning a vacant piece of level ground. The origin further led to an irregular derivation of area as a particular amount of space contained within a set of boundaries. The use of area has many practical applications in building, farming, architecture, science.We need to find the area of the room floor to determine the size of the carpet to be bought. Covering the floor with tiles, covering the wall with paint or wallpaper or building a swimming pool are other examples, where in the area is computed. In real life, not every plane figure can be clearly classified as a rectangle, square or a triangle. To find the area of a composite figure which consists of more than one shape, we need to find the sum of the area of both or all the shapes forming the composite figure. Area is a measure of how much space there is on a flat surface. Different shapes have different ways to find the area. For example, in a rectangle we find the area by multiplying the length times the width, for square area is A = S x Swhere “S” represents the length of one side and for trianglearea isA = ½ x B x H “B” is the base of the triangle, and “H” is the height .

d) Speed and Time:

Speed, distance, and time refer to the three important variables in the field of Science (physics). These three variables certainly facilitate the solving of several types of problems not only in science but also in mathematics. Speed refers to the measure of how quickly an object moves from one point to another. The speed of any particular object refers to the magnitude of its velocity. It is a scalar quantity. Units of speed are m/s , cm/s etc. Distance refers to the amount of space which exists between any two points. It can also be explained as the amount something has moved. Units of distance are like miles, kilometers, meters, centimeters, etc. . Moreover, the distance in which something travels has a relation with the change in position. Time refers to the progression of events. This progression is in such a manner that it goes from the past to the present and into the future. Therefore, if there is a system which is unchanging in nature, then it is timeless. Furthermore, time is not something that one can see, touch, or taste. One can only measure its passage. Scientists believe the time to be the fourth dimension of reality. Time is measured in Hours, minutes, Seconds etc.

Speed Distance Time Formula

There certainly exists direct relationship between the three important variables of speed, distance, and time. The speed distance time formula demonstrates the relationship between these three variables. First of all, speed is directly related to the other two variables of distance and time. This is because, speed refers to distance divided by time and it expression is below:

Speed = distance/ time, s = d/ t

Furthermore, one can ascertain the relationship of time with the other two variables by dividing the distance with speed. Its expression is as follows:

Time = distance/speed, t = d/s

Finally, to find the distance, speed is beside time. Therefore, distance certainly is speed multiplied by time.

Distance = speed × time, d = s t

e) Volume:

Volume is the amount of space that a substance or object occupies, or that is enclosed within a container. Volume is defined as the amount of space taken up by an object. The volume of an object is measured in cubicunits such as cubic meters, cubic inch, cubic foot, cubic meter, etc.With liquids we use different units, but the concept is the same. Liquid volumes have units like litres, gallons, pints, and milli litres etc. One way to find the volume of an object is to completely submerge the object in water and measure the volume of water that is displaced by the object.Volume is a measure of how much space an object takes up. For example two shoe boxes together have twice the volume of a single box, because they take up twice the amount of space. Volume is the quantity of three-dimensional space enclosed by a closed surface, for example, the space that a substance (solid, liquid, gas, or plasma) or shape occupies or contains. The volume of a container is generally understood to be the capacity of the container; i. e., the amount of fluid (gas or liquid) that the container could hold, rather than the amount of space the container itself displaces. Three dimensional mathematical shapes are also assigned volumes. Volumes of some simple shapes, such as regular, straight-edged, and circular shapes can be easily calculated using arithmetic formulas. Volumes of complicated shapes can be calculated with integral calculus if a formula exists for the shape's boundary. One-dimensional figures (such as lines) and two-dimensional shapes (such as squares) are assigned zero volume in the three-dimensional space. Determining the volume of an object can be of practical importance. Knowing the volume of an object can help you determine how much would be needed to fill that object, like the amount of water needed to fill an aquarium. Understanding volume or capacity is especially important in the field of medicine or chemistry when one is dealing constantly with liquid measurement. Different shapes have different ways to find the volume. For example, in a cube we find the volume by multiplying the three side lengths together.

f) Surface Area:

Surface area of three-dimensional solids refers to the measured area, in square units, of all the surfaces of objects like cubes, spheres, prisms and pyramids. It is the area covered or region occupied by the surface of that particular object. It is the total area of the surface of a three-dimensional object. It is the sum of all of the areas of each of the faces.Surface area is the amount of space covering the outside of a three-dimensional shape. It is commonly denoted by S for a surface in three dimensions, or A for a region of the plane. It is a two-dimensional property of a three-dimensional figure. The surface area is the area that describes the material that will be used to cover a geometric solid. When we determine the surface areas of a geometric solid we take the sum of the area for each geometric form within the solid. For every 3D solid, we can examine each face or surface and calculate its surface area. Then, adding all the individual surface areas, we can find the surface area of the entire solid. For example the surface area of a cube is the area of the six squares that cover it. The area of one of them is a* a, or a 2. Since these are all the same, we can multiply one of them by six, so the surface area of a cube is 6 times one of the sides squared.Shapes with curved surfaces too have surface area.The cylinder can be thought of as two circles at each end and a strip wrapped around to form the body. The two circles have an area of π r2. The strip is a rectangle of width h, and a length equal to the circumference of each circle

($2\pi r$). So again we can determine the total surface area as the sum of the two circles and the rectangular strip. An understanding of surface area is important to the chemist because chemical reactions occur between particles on the surface of the bulk of mass. Therefore the surface area of a sphere is of interest to the chemist, making the assumption that particles are spherical in shape. Surface area is alsoimportant in chemical kinetics. Increasing the surface area of a substance generally increases the rate of a chemical reaction. For example, iron in a fine powder will combust, while in solid blocks it is stable enough to use in structures. Also in Biology the surface area of an organism is important in several considerations, such as regulation of body temperature and digestion.

Check Your Progress

Notes :

(a) Write your answers in the space given below.

(b) Compare your answers with those given at the end of the unit.

3. Mathematics Club should celebrate days and events pertaining to the history of mathematics or great mathematicians. True/False

....

....

4. What do you mean by Area?

....

....

5. What do you mean by Speed?

....

....

10.9 LET US SUM UP

Mathematics club can have a number of different objectives, but their essential purpose is to supplement the mathematics education provided for children and young people in the formal system where the process of adapting school syllabuses to new trends in mathematics teaching is often slow. The activities of mathematics club can cover a wide variety of topics relating to the subject. If the students participate in such activities whole-heartedly, then we derive great benefit. The club can go a long way in arousing and maintaining interest of the students. They will develop love for the subject. The utility of mathematics club depends upon the interest shown by the teacher and the extent to which the students are motivated to take part in the activities of mathematics club.

10.10 LESSON END EXERCISE

1. Discuss various objectives of Mathematics Club?
2. Explain in detail the importance of Mathematics club?
3. Discuss the different activities of club in respect to teaching of circle to the students?

10.11 SUGGESTED FURTHER READING

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10.12 ANSWERS TO CHECK YOUR PROGRESS

1) (a) To Encourage a positive attitude towards mathematics

   (b) To arouse and maintaining students' interest in mathematicsExpress Stage

2) True

3) True

4) Area is a mathematical term defined as the two-dimensional space taken up by an object. It is the space occupied by a flat shape or the surface of an object. The area of a figure is the number of unit squares that cover the surface of a closed figure

5) Speed refers to the measure of how quickly an object moves from one point to another. The speed of any particular object refers to the magnitude of its velocity. It is a scalar quantity.

—×—

LESSON NO. 11 **UNIT-IV**

QUALITIES OF MATHEMATICS TEACHER

STRUCTURE

11.1 INTRODUCTION

NCF (2005) emphasisedthat main goal of teaching mathematics is the mathematisation of the child's thought processes. For achieving this goal we needmathematics teachers who possess certain skills and qualities pertaining to the subject. Academic qualifications and qualities matter a lot in mathematics teaching. Mathematics teacher helps students to make a successful to college and prepare them for career in high demand fields related to science, technology and engineering.The Present chapter may help in understanding the qualities of mathematics teachers, Competencies of mathematics teachers- listening, understanding and expression.

11.2 OBJECTIVES

After you study this lesson, you will be able to:

1) Describe various general qualities of mathematics teacher
2) Describe various special qualities of mathematics teacher
3) Explain the competencies of mathematics teacher

11.3 QUALITIES OF MATHEMATICS TEACHER

People have divergent opinions regarding how to identify the qualities of a great math teacher. However, one thing that remains unchanged is that all great math teachers have certain qualities that distinguish them from others. These qualities make them produce great results not only for the students but also for themselves. A mathematics teacher apart from being teacher of a particular subject is essentially a teacher at first. Therefore the qualities of a mathematics teacher may be divided into two categories:

A) General qualities and abilities that are required in every teacher
B) Special qualities that are expected from a good mathematics teacher

11.3.1 GENERAL QUALITIES OF MATHEMATICS TEACHER

1) He should be friend ,philosopher and guide
2) He should be neat and well-groomed
3) He should be firm but kind
4) He should be organised, stimulating and enthusiastic in his work
5) He should be polite, courteous, tolerant and open-minded.

6) He shouldhave a sense of humour so that he can able to make subject interesting.

7) He should have a love for discipline.

8) He should have qualities of leadership.

9) He should be good motivator.

10) He should have a caring attitude towards their students and is always ready to support those who are having problems.

11) He should have knowledge of individual differences because every students is different in reading, writing, understanding and work speed in mathematics.

12) He should be ready to work hard & putting best effort for lecture preparation as per different courses and level of students.

13) He should have a lot of patience with slow learners, forgiveness to mischievous students.

14) He should have ability to engage class and create a learning environment.

15) He should have courage to handle challenging problems.

16) He should interact with students both in and out of the classroom.

17) He should create learning environments where students are active participants as individuals and as members of collaborative groups.

18) He should have effective communication skills.

19) He should be impartial and fair.

20) He should be very Punctual.

21) He should have an optimistic attitude about learning and about the students.

22) He should believe that all students could be successful.

23) He should be able to control the classroom properly.

24) He should be precise and objective.

25) He should be open to critics.

26) He should have incredible working ethic.

27) He should be cooperative and understanding.

28) He should be objective in assessing students' work.

29) He should be professional. Professional attitude in this case is defined as an attitude of doing jobs as well as possible and having high competence.

30) He should have adequate expression power.

31) He should have adequate general knowledge.

32) He should have undergone a suitable course of teacher preparation like ETT, B.Ed.

33) He should have a habit of self-study.

34) He should have satisfactory mental health and adequate adjustment.

35) He should have satisfactory relationship with parents of students and members of community.

36) He should help the students in meeting their psychological needs.

37) He should take care of his professional development by attending various seminars and workshops with open mind to learn.

11.3.2 SPECIAL QUALITIES OF MATHEMATICS TEACHER

1) Sound Knowledge of Mathematics

Mathematics teacher should have an extensive understanding of mathematics. He should undergo a thorough training process in a recognized college or university where he acquires the knowledge and skills needed to teach learners effectively. The knowledge he get from these institutions give him the confidence to explain clearly all the mathematical concepts to hisstudents.

2) Knowledge of Teaching Strategies

Mathematics teacher should have knowledge of different teaching strategies of mathematics. He should keep up with the best practices in mathematics education and regularly incorporates them into his instructions to help all of his students learn. He also understands there may be multiple ways to solve problems and uses those alternate strategies to help struggling students grasp difficult concepts.

3) Facilitator

Mathematics teacher should not force students to follow his own approach. He should acts as facilitator, allowing the students to offer suggestions. He should provide students with the knowledge and tools to solve problems and then encouraging students to solve them on their own. He shouldallow room for collective discussions so that everyone in the class arrives at the same conclusion. He should provide the right guidance and support where necessary.

4) Constantly Learning

Mathematics teacher should understand that he is not perfect. That is why he should constantly read new materials to update his knowledge base. He should also try to enrol himself for supplementary courses in his areas of specialty to better him and gain more confidence in the classroom. This may involve unlearning the outdated algorithms and mathematical terms and learning new ones. Once a great teacher learns about a new mathematical concept, he lets all the students know about it, leading to effective learning and better grades. In order to enrich his knowledge he should study mathematical journals and other books of professional interest.

5) Strong Interest in mathematics

A mathematics teacher should have strong interest in mathematics

6) Positive attitude towards mathematics

A mathematics teacher should have positive attitude towards his teaching subject. Because his self-attitude directly influence the learning process of the students.

7) Knowledge of different teaching methods-

With a firm grip over the subject matter, a mathematics teacher should know the teaching methods. He should clearly know the aims and objectives of mathematics.

8) Presentation of subject matter

Mathematics teacher should present the subject matter skillfully making interaction with the students, introducing well methods and applying various aids

9) Inspiration to drill work

A mathematics teacher should inspire and motivate the students to drill and practice works through examples understand the basic concept of formulae.

10) Participate enthusiastically in various activities related to Mathematics

Mathematics teacher should participate enthusiastically in various school activities related to mathematics such as organising mathematics club, setting up mathematics laboratory, Students mathematics Projects etc.

11) Handling different types of students

He should have ability to motivate brilliant students to keep on practicing and dive more into mathematics. At the same time, helping happily slow learners with basic things and remove their fear/ phobia for mathematics and keep a balance between the two

12) Help students to develop mathematical thinking.

He should make students learn how to do math, instead to giving them solution to particular problem. At the end they should be able to do any new problem on the same topic.

13) Engage students in reasoning and problem solving

He should understandthat idealmathematics learning is not apassive process of memorizing and using standard algorithms, instead students should be engaged inreasoning, problem solving, and encouraged for discourse with teachers and other students to exploremathematical problems.

14) Creative

Mathematics teacher should be creative and have through knowledge of mathematical creativity.He should be able to cultivate originality and creativity among his students for the solution of mathematics problem. He should try to inculcate mathematical creativity among his students.

15) He should be able to associate mathematics with everyday life.

16) He should be able toconvey the beauty and elegance of mathematics to his students.

17) He should be happy and enthusiastic to teach mathematics.

18) He should have the knowledge of learning outcomes in mathematics.

19) He should have Love for mathematics & passion for teaching.

20) In addition to his knowledge of mathematics, he should have background knowledge of other fields as well.

21) He should do professional research work and must write professional articles in different journals of mathematics.

22) He should have the knowledge of history of mathematics and contributions and life history of great mathematicians.

23) He should possess the essential mathematical skills e.g problem solving skill, skill of computation, drawing and sketching, reading the log tables, working with calculator and computers etc.

24) He should have knowledge of different evaluation techniques and able to prepare, administer and score the different achievement tests, diagnostic tests in mathematics. He should use a range of assessment procedures which reflect different approaches to teaching and learning.

25) He should be very resourceful. He should be able to use his resourcefulness in the mathematics class whenever required.

26) He should know about the availability of different audio-visual aids in the teaching of mathematics and their proper utilization. He should use the full range of available and appropriate technology.

27) He should develop heuristic attitude among his students. He should encourage and invite pupils to put him questions and treat them with sympathetic attitude. He should encourage inquiry, reflective thinking and creative expressions in his class.

28) He should be able to assign suitable homework to his students according to their mental level. He should give only that assignment whichare clear and understood by the students.

29) He should be serious in accomplishing his objectives.

30) He should check the errors committed by his students and plan remedial instructions accordingly.

31) He should foster concretization of abstract concepts of mathematics.

32) He should encourage students to expand their mode of communication in Mathematics.

33) He should convey the wholeness of Mathematics rather than presenting it as a disjointed collection of topics.

34) He should encourage students to learn together in cooperative small groups.

35) He should invoke the power of visual imagery and mental arithmetic among his students.

36) He should recognize the key role of parents in the student's development.

37) He should displays effective and efficient classroom management that includes classroom routines that promote comfort, order and appropriate student behaviours.

38) He should effectively allocate time for students to engage in hands-on experiences, discuss and process content and make meaningful connections.

39) He shouldcreate an environment where student work is valued, appreciated and used as a learning tool.

40) He should uses student work/data, observations of instruction, assignments and interactions with colleagues to reflect on and improve teaching learning process.

41) He should provide regular and timely feedback to students and parents that move learners forward.

42) He should work with other teachers to make connections between and among disciplines.

43) He should use and promote the understanding of appropriate content vocabulary in mathematics.

Check Your Progress

Notes :

(a) Write your answers in the space given below.

(b) Compare your answers with those given at the end of the unit.

1. Mathematics teacher should ——————————concretization of abstract concepts of mathematics

....

....

2. Mathematics teacher should be very punctual True/False

....

....

11.4 COMPETENCIES OF MATHEMATICS TEACHER:

To possess a competence in some domain of personal, professional or social life is to master essential aspects of life in that domain. Mathematical competence means the ability to understand, judge, do, and use mathematics in a variety of intra- and extra-mathematical contexts and situations in which mathematics plays or could play a role. Necessary, but certainly not sufficient, prerequisites for mathematical competence are lots of factual knowledge and technical skills, in the same way as vocabulary, orthography, and grammar are necessary but not sufficient prerequisite for literacy. Competencies of mathematics teachers are particular qualities that must be possessed by him to be great mathematics teacher. Competencies are not skills, although they are similar. Skills are learned, while competencies are inherent qualities an individual possesses – combining skills, knowledge and ability.

The core competencies that should be possessed in mathematics teacher include.

1) **Mathematical Thinking** :

- He should be able to pose questions that are characteristic of mathematics, and know the kinds ofanswers that mathematics may offer.
- He must understand and handle the scope and limitations of a given concept.
- He should be able to extend the scope of a concept by abstracting some of its properties; generalising results to larger classes of objects.
- He should be able to distinguish between different kinds of mathematical statements (includingconditioned assertions ('if-then'), quantifier laden statements, assumptions, definitions, theorems, conjectures, cases).

2. **Ability to Pose andsolve mathematical problems**

- He should be able to identify, pose, and specify different kinds of mathematical problems.
- He should be able to solve different kinds of mathematical problems whether posed by others or by himself, and, if appropriate, in differentways.

3. **Modelling mathematically**

- He should be able to analyse foundations and properties of existing models, including assessing theirrange and validity.
- He should able to decode existing models, i.e. translating and interpreting model elements interms of the 'reality' modelled.
- He should be able to perform active modelling in a given context.
- He should be able to structure the field.
- He should be able to validate the model, internally and externally.
- He should be able to analyse and criticise the model, in itself and vis-à-vis possible alternatives.
- He should be able to communicate about the model and its results.
- He should be able to monitor and control the entire modelling process.

4. **Mathematical Reasoning**

- He should be able to follow and assess chains of arguments, put forward by others.
- He should be able to know what a mathematical proof is , and how it differs from other kindsof mathematical reasoning, e.g. heuristics.
- He should able to uncover the basic ideas in a given line of argument (especially a proof), including distinguishing main lines from details, ideas from technicalities.

- He should be able to devise formal and informal mathematical arguments, and transforming heuristic arguments to valid proofs, i.e. proving statements.

5. Representing mathematical entities (objects and situations)

- He should be able to understand and utilise (decoding, interpreting, distinguishing between) different sorts of representations of mathematical objects, phenomena and situations.
- He should be able to understand and utilise the relations between different representations of thesame entity, including knowing about their relative strengths and limitations.
- He should be able tochoose and switch between representations.

6. Handling mathematical symbols and formalisms

- He should be able to decode and interpret symbolic and formal mathematical language, and understand its relations to natural language.
- He should be able to understand the nature and rules of formal mathematical systems (both syntaxand semantics)
- He should be able to translate from natural language to formal/symbolic language.
- He should be able to handle and manipulate statements and expressions containing symbols andformulae.

7. Communicating in, with, and about mathematics

- He should be able to understand others' written, visual or oral 'texts', in a variety of linguistic registers, about matters having a mathematical content.
- He should be able to express himselfat different levels of theoretical and technical precision, inoral, visual or written form, about such matters.

8. **Making use of aids and tools(including IT)**

- He should be able to know the existence and properties of various tools and aids for mathematical activity, and their range and limitations.
- He should be able to reflectively use such aids and tools.

11.5 LISTENING

An effective mathematics teacher is not only a knowledgeable and skilled teacher but he is also a good active listener. Good listening skills are needed to develop empathy and understanding with the students and to assess whether they understand what they are being taught. Listening skills also help in negotiating with students and defusing any potential classroom conflicts. Listening is a two-way process: we, as the teachers do most of the talking but we must also learn to listen to the students; the students spend a lot of time listening to us, and will also benefit from improved listening skills. Both teacher and student must learn to respect each other and that could be possible only if both are good listeners. Mathematics teacher should open to himself to the 'incoming message' by letting down his defences as far as possible, and trying to sense the real, underlying meaning of what is being said. He must listen for ideas, implications and feelings, as well as the facts being conveyed. He should interpret, or reconstruct, what is being said, remembering always that words have different meanings to different people. He should keep asking himself whether he really understand the message. He should do his best to listen with full attention, and withhold judgement, assumption and criticism at early stage. He should not jump to conclusions before the story is complete. He should allow the students to finish their message before attempting to begin speaking and reaching conclusion about what they have said. He should evaluate what is being said, only after he has made a reasonably objective interpretation of the message of speaker. At this point he should reflect on the information and options being presented, and sift through the evidence. Unfortunately, judging often starts far too early in the listening process. It is a fact that many people will judge according to their own personal life experiences and this may have a negative implication on the message. Unskilled listeners close their ears to words they do not want to hear and only hear the words they want to hear. Teacher should also be good responder. Teacher should demonstrate that he has truly been listening. He should reassure the speaker that he has been giving him full attention is a critical aspect of constructive listening. Teacher should provide feedback by asking for clarification or for more information, or at least giving some visible acknowledgment by smiling, nodding or frowning. Teacher should be aware that all the people in the classroom, including himself, are filtering and interpreting every word through a personal screen of attitudes, values, assumptions, judgements, past experiences and strong feelings. He should be aware too that listening behaviour will be influenced by factors such as age, sex, cultural background and even physical appearance and mannerisms.

Some of the ways teachers can convey the genuine desire to understand are:

He should be attentive, alert and not easily distracted.

He should create a positive atmosphere with his non-verbal behaviour

He should have appropriate body language and facial expressions.

He should be interested in the students' needs.

He should listen in a friendly way.

He should be non-judgmental and should not criticise.

He should respect privacy and not ask intrusive or complicated questions.

He should act like a mirror and reflect what he think is being felt and said.

He should show that he is in no hurry and should remember that silences throughout teaching are good, as they give students opportunities to think and reflect on questions and topics in their mind before verbally giving an answer.

He should not negate any aspect of a problem, even if it seems unimportant to him. To a student, it may be crucial to their learning.

He should not get emotionally involved, angry, upset or argumentative. He needs to remain professional in his interactions with students, as a role model and the students are looking up to him for guidance and direction.

He should not jump to conclusions or judgements about any students.

He should not have any pre-conceived ideas or notions about any student based on what he may have heard from another colleague or former teacher.

Ways to indicate that Teacher is listening:

He should give encouraging acknowledgements (eg. "Yes" or "I see" or nodding or "Ah ha").

He should give non-verbal acknowledgements (eg. relaxed body posture, eye contact, facial expression. Remember that people can speak with their bodies without saying a solitary word; a movement can indicate a great deal about how a person is feeling)

He should invite more responses (eg. 'Tell me more' or 'I'd like to hear about that' – these few words can imply you are keen for the student to expand on their message because it has relevance to you and the rest of the group).

Teacher should not do for group listening:

He should not interrupt

He should not change the subject

He should not rehearse in his head instead of listening

He should not interrogate

He should not teach or preach

He should not give any unnecessary advice

He should not talk down to students. They can sense when the teacher is not on their level and may not respond appropriately

11.6 UNDERSTANDING

Mathematics teacher must understand twothings; one thing is to study whom he is teaching, the other thing is to study the knowledge he is teaching.If he can interweave the two things together nicely, he will succeed as a teacher. He must understand that he must interpret students' written work, analyse their reasoning, and respond to the different methods they might use in solving a problem. Teaching requires the ability to see the mathematical possibilities in a task, sizing it up and adapting it for a specific group of students. Familiarity with the trajectories along which fundamental mathematical ideas develop is crucial if a teacher is to promote students' movement along those trajectories. He must understand that he needs to master and deploy a wide range of resources to support the acquisition of mathematical proficiency. Three kinds of understanding are crucial for teaching school mathematics: Understanding mathematicalknowledge, Understanding his students, and Understandingof instructional practices.

Understanding Mathematical knowledge : It includes knowledge of mathematical facts, concepts, procedures, and the relationships among them; knowledge of the ways that mathematical ideas can be represented; and knowledge of mathematics as a discipline—in particular, how mathematical knowledge is produced, the nature of discourse in mathematics, and the norms and standards of evidence that guide argument and proof. In our use of the term, knowledge of mathematics includes consideration of the goals of mathematics instruction and provides a basis for discriminating and prioritizing those goals. Knowing mathematics for teaching also entails more than knowing mathematics for oneself. Mathematics teacher certainly need to be able to understand concepts correctly and perform procedures accurately, but he also must be able to understand the conceptual foundations of that knowledge. In the course of his work as a teacher, hemust understand mathematics in ways that allow him to explain and unpack ideas in ways not needed in ordinary adult life. The mathematical sensibilities he holds matter in guiding his decisions and interpretations of students' mathematical efforts.

Understanding his students: It includes both knowledge of the particular students being taught and knowledge of students' learning in general. Knowing one's own students includes knowing who they are, what they know, and how they view learning, mathematics, and themselves. The teacher needs to know something of each student's personal and educational background, especially the mathematical skills, abilities, and dispositions that the student brings to the lesson. The teacher also needs to be sensitive to the unique ways of learning, thinking about, and doing mathematics that the student has developed. Each student can be seen as located on a path through school mathematics, equipped with strengths and weaknesses, having developed his or her own approaches to mathematical tasks, and capable of contributing to and profiting from each lesson in a distinctive way. he not only must understand his students but also how they learn mathematics. It includes general knowledge of how various mathematical ideas develop in children over time as well as specific knowledge of how to determine where in a developmental trajectory a child might be. It includes familiarity with the common difficulties that students have with certain mathematical concepts and procedures, and it encompasses knowledge about learning and about the sorts of experiences, designs, and approaches that influence students' thinking and learning.

Understanding of instructional practice: It includes knowledge of curriculum, knowledge of tasks and tools for teaching important mathematical ideas, knowledge of how to design and manage classroom discourse, and knowledge of classroom norms that support the development of mathematical proficiency. Understanding norms that support productive classroom activity is different from being able to develop and use such norms with a diverse class.Understanding what is to be taught and how to plan, conduct, and assess effective lessons on that mathematical content. It includes knowledge of learning goals as expressed in the curriculum and knowledge of the resources at one's disposal for helping students reach those goals. It also includes skill in organizing one's class to create a community of learners and in managing classroom discourse and learning activities so that everyone is engaged in substantive mathematical work.

11.7 EXPRESSION

Mathematics teacher should have a good expression. It is the act of saying what he thinks or showing how hefeels using words or actions. He should express his ideas/concept in such a way that each and every student understands it. Mathematics teacher should be able to express the following things effectively to his students

1. He should be able to express mathematical principles, concepts, processes, algebraic quantities etc.
2. He should be able to express various logarithm and other tables of mathematics.
3. He should be able to express various statistical graphs.
4. He should be able to express mathematical language and symbol.
5. He should able to express the contribution of mathematicians.
6. He should be able to express mathematical literature.
7. He should be able to express inter relationship and interdependence of different branches of mathematics.
8. He should be able to express the signification of different units of measurement.
9. He should be able to express various geometric figures.
10. He should be able to express the importance of speed and accuracy in performing arithmetical computations and operations.
11. He should be able to express various observational skills e.g reading graphs, surveying and weighing.
12. He should be able to express the technique of problem solving.
13. He should be able to express the ability to apply the mathematical knowledge in other fields and subjects.

14. He should be able to express his love for mathematics and develops the same in his students.

15. He should be able to express critical attitude in observation and thought.

16. He should be able to express the power of logical thinking and concentration.

17. He should be able to express the attitude of truthfulness in observation and drawing conclusions based on accurate facts.

18. He should be able to express the attitude of respect for other point of view and ready to change own decision on presentation of new and convincing facts.

19. He should be able to express unbiased and impartial attitude in judgments.

20. He should be able to express ways for planned and systematic procedure in solving a problem.

21. He should be able to express the ability to organize mathematics exhibitions and fairs etc.

22. He should be able to express the ability to discuss and argue using mathematical terminology.

23. He should be able to express the ability to construct, improvise and manipulate mathematical equipment and models.

24. He should be able to express the contribution of mathematics in the progress of civilization and culture.

25. He should be able to express the role of mathematics in everyday life.

26. He should be able to express the aesthetic value of mathematics by observing symmetry, similarity, order and arrangements in mathematical facts, principles and process.

27. He should be able to express the vocational values, recreational value and amusement value of mathematics.

28. He should be able to express his love for mathematics literature, mathematics hobbies, nature and mysteries of mathematics, mathematics club, mathematics laboratory etc.

Check Your Progress

Notes :

(a) Write your answers in the space given below.

(b) Compare your answers with those given at the end of the unit.

3. What is mathematical Competence?

....

....

4. Mathematics teacher should understand what is to be taught and how to plan, conduct, and assess effective lessons on that mathematical content..........True/false

....

....

5. How does teacher indicate that he is listening?

....

....

11.8 LET US SUM UP

Mathematics education relies very heavily on the mathematics teacher. Therefore he should have his own understanding of mathematics, of the nature of mathematics, and of pedagogic techniques. It is the zeal, enthusiasm, sincerity towards work and devotion to his profession which matter most. The most effective mathematics teachers are those, who are able to grow not only in knowledge of their subject but in their understanding of life both in and out of class-room. For them the classroom is fascinating laboratory of life wherein they grow in their ability to understand more about their subject, more about children and more about themselves as teacher and as individuals.

11.9 LESSON END EXERCISE

1. Discuss various general qualities of mathematics teacher?
2. Explain in detail special qualities of mathematics teacher?
3. Discuss listening as a competency of mathematics teacher?

11.10 SUGGESTED FURTHER READING

Burner, J.S., (1961); the Process of Education, Harvard University Press, Combridge.

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NCTM.(1989). Curriculum and evaluation standards for school mathematics. Reston, VA: Author.

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The National Council of Teacher of Mathematics, (1953); The Learning of Mathematics, Twenty-first Year Book, Washington D.C., U.S.A.

11.11 ANSWERS TO CHECK YOUR PROGRESS

1) Foster
2) True

3) Mathematical competence means the ability to understand, judge, do, and use mathematics in a variety of intra- and extra-mathematical contexts and situations in which mathematics plays or could play a role.

4) True

5) He should give encouraging acknowledgements (eg. “Yes” or “I see” or nodding or “Ah ha”).

   He should give non-verbal acknowledgements and he should invite more responses.

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